Name (Last, First) _____ **ID #** _____

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Lecturer _____ **Section (01, 02, 03, etc.)** _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 2** **November 14, 2018**
7:00-9:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are nine (9) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do **not** use a calculator, reference materials, or paper other than a booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	11	
2	11	
3	11	
4	11	
5	11	
6	11	
7	11	
8	11	
9	11	
Free	1	1
TOTAL	100	

#1. Find the derivatives of the following functions. You do NOT need to simplify your answer.

(a) (3 points) $f(x) = 2x^3 + \dfrac{1}{2x^3} + 2(3^x) + 2(3^{-1})$.

(b) (4 points) $g(x) = \dfrac{\cos(5x)}{5 - e^{5x}}$

(c) (4 points) $h(x) = \sqrt{x^{-3}\sec(x)}$

#2. Let A be the area of a circle and let r be its radius. At a certain instant, $A = 36\pi$ square meters and $\dfrac{dA}{dt} = 9$ square meters per second.

(a) (6 points) Find $\dfrac{dr}{dt}$ at this instant. Your answer may involve the number π.

(b) (5 points) Let P be the circumference of the circle. Find $\dfrac{dP}{dt}$ at this instant. Your answer may involve the number π.

#3. Consider the curve $x^2 + (y-1)^3 = 9$.

(a) (6 points) Use implicit differentiation to find $\dfrac{dy}{dx}$. Your answer may be an expression involving x and y.

(b) (5 points) Find the equation of the tangent line to the curve at the point where $x = 1$.

#4. Let $f(x) = \ln((x-3)^4 + 16)$.

(a) (6 points) Find all the critical numbers of $f(x)$.

(b) (5 points) Find the absolute maximum and minimum of $f(x)$ on the interval [2,5]. Your answer may use logarithms.

#5.

(a) (6 points) Let $f(x) = x^{77} + 77x^{50} + 50$.
Is there a number c in the interval (40, 41) such that the *instantaneous* rate of change of f with respect to x at c is equal to the *average* rate of change of $f(x)$ with respect to x over the interval [40, 41]? Justify your answer by citing any relevant theorems and why they apply.

(b) (5 points) Let $g(x) = x^2$. Suppose that the conclusion of the Mean Value Theorem is satisfied for $g(x)$ by $c = 3$ and the interval $[2, b]$. What is the value of b?

#6. Element X and Element Y are two different radioactive elements. A sample of each element decays at a rate proportional to the amount of mass in the sample. The decay rates for Element X and Element Y are not the same.

(a) (6 points) A sample of Element X has an initial mass of 10 grams. After exactly 20 days the sample has a mass of 5 grams. Find an expression for the mass $m(t)$ of the sample after t days. Your answer may involve logarithms and the number e.

(b) (5 points) A sample of Element Y takes exactly 30 days to decay from m_0 grams to $\frac{m_0}{3}$ grams. Find how long it would take for a sample of Element Y to decay from m_0 grams to $\frac{m_0}{4}$ grams, or state that more information is needed to solve the problem. Justify your answer. Your answer may involve logarithms and the number e.

#7. (14 points) A table of values for the the functions $f(x), g(x)$ and $h(x)$ and their first two derivatives is given below. You may assume that $f(x), g(x)$ and $h(x)$ and their first two derivatives exist at all real numbers. Justify your answers.

x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$	$h(x)$	$h'(x)$	$h''(x)$
1	4	8	7	8	4	6	9	3	5
2	6	5	3	1	3	2	9	2	-5
3	1	4	-8	2	1	3	2	1	0

(a) (3 points) Let $P(x) = f(x)g(x)h(x)$. Find $P'(1)$.

(b) (4 points) Let $Q(x) = \dfrac{f(x)}{g(x)}$. Find $Q''(2)$.

(c) (4 points) Let $R(x) = g(h(x))$. Find $R'(3)$.

#8.

(a) (6 points) Let $f(x) = (\sin x)^{\cos x}$. Use logarithmic differentiation to find $f'(x)$ as a function of x. You do NOT need to simplify your answer.

(b) (5 points) Let $g(x) = \arctan(1 + \tan(x))$. Find $g'(x)$. You do NOT need to simplify your answer.

#9. Let $f(x) = \sin(\pi x)$.

(a) (6 points) Find the linearization $L(x)$ of $f(x)$ at the point $a = \frac{1}{6}$. Your answer may involve the number π and square roots.

(b) (5 points) Use your answer in part (a) to find an approximate value for $\sin(\frac{\pi}{12})$. Your answer may involve the number π and square roots.

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