**Name (Last, First)** _____ **ID #** _____

**Signature** _____

**Lecturer** _____ **Section (01, 02, 03, etc.)** _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

**Math 131** **Exam 1** **October 3, 2018**
**7:00-9:00 p.m.**

**Instructions**

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are seven (7) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do **not** use a calculator, reference materials, or paper other than a booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

| QUESTION | PER CENT | SCORE |
|---|---|---|
| 1 | 14 | |
| 2 | 14 | |
| 3 | 14 | |
| 4 | 14 | |
| 5 | 14 | |
| 6 | 14 | |
| 7 | 14 | |
| Free | 2 | |
| TOTAL | 100 | |

#1. Find all vertical and horizontal asymptotes of the following functions, if any. Justify your answers using limits.

(a) (7 points) $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 5x + 6}$

(b) (7 points) Suppose $h(x)$ is a function such that $-x \leq h(x) \leq x^2 - 3x + 1$. Evaluate $\lim_{x \to 1} h(x)$ or state that more information is needed to evaluate it.

#2. (a) (7 points) Define the function $f(x)$ by

$$f(x) = \begin{cases} ax^2 + 3x & x < 1 \\ b & x = 1 \\ x^3 - ax & x > 1 \end{cases}$$

For what values of $a$ and $b$ is $f(x)$ continuous at $x = 1$? Justify your answer.

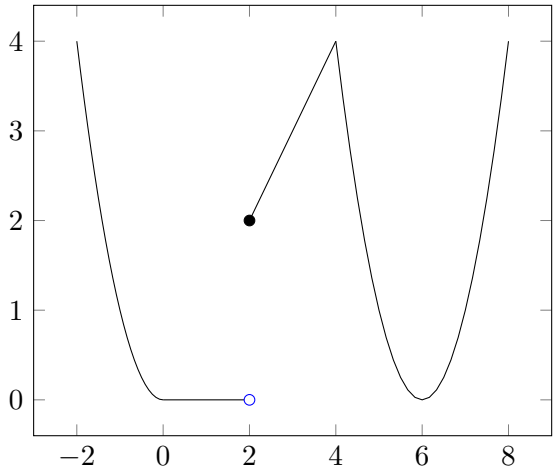
(b) (7 points) Define the function $g(x)$ by

$$g(x) = \begin{cases} x & x < 1 \\ 2x - 1 & x \geq 1 \end{cases}$$

Is $g(x)$ differentiable at $x = 1$? Justify your answer using the limit definition of derivative.

#3. The definition of a function $f(x)$ and a portion of its graph are shown below.

$$f(x) = \begin{cases} x^2 & x < 0 \\ 0 & 0 \le x < 2 \\ x & 2 \le x < 4 \\ (x-6)^2 & x \ge 4 \end{cases}$$




(For $(a), (b)$, and $(c)$ you may state your answers without justification.)

(a) (5 points) Find all the values $x$ at which $f(x)$ is discontinuous.

(b) (5 points) Find all the values $x$ at which $f(x)$ is not differentiable.

(c) (4 points) Evaluate $\lim_{x \to 2^+} f(f(x))$.

#4. The cost (in dollars) of producing $x$ cars is $C(x) = x^2 + 3x + 1$.

(a) (5 points) Find the average rate of change of $C$ with respect to $x$ when production is raised from $x = 4$ to $x = 5$.

(b) (5 points) Find the instantaneous rate of change of $C$ with respect to $x$ when $x = 5$. Justify your answer using limits.

(c) (4 points) Find the equation of the tangent line to the graph of $C$ at $x = 5$.

#5. Let $f(x) = 7x - 7$.

(a) (8 points) Find the largest value of $\delta$ such that if $|x-2| < \delta$, then $|f(x)-7| < \epsilon$. Express your answer in terms of $\epsilon$.

(b) (6 points) If we require $x$ to be in the interval $(2.5, 3.5)$, what is the smallest value of $\epsilon$ such that $|f(x) - 14| < \epsilon$?

#6. (14 points) Let $g(x) = 3^{-x}$ and $h(x) = x$. Prove that there is some number $c$ such that $g(c) = h(c)$. Precisely state any theorems you use.

#7. (14 points) Use the limit definition of the derivative to find $f'(a)$ for

$f(x) = \sqrt{x^2 + 1}$.

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