Instructions

• Turn off all cell phones and watch alarms! Put away iPods, etc.

• There are seven (7) questions.

• Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.

• Do not use a calculator, reference materials, or paper other than a booklet.

• Organize your work in an unambiguous order. Show all necessary steps.

• Answers given without supporting work may receive 0 credit!

• Be ready to show your UMass ID card when you hand in your exam booklet.

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#1. Find all vertical and horizontal asymptotes of the following functions, if any. Justify your answers using limits.

(a) (7 points) \( f(x) = \frac{x^2 - 3x + 2}{x^2 - 5x + 6} \)

(b) (7 points) Suppose \( h(x) \) is a function such that \(-x \leq h(x) \leq x^2 - 3x + 1\). Evaluate \( \lim_{x \to 1} h(x) \) or state that more information is needed to evaluate it.
#2. (a) (7 points) Define the function $f(x)$ by

$$f(x) = \begin{cases} 
ax^2 + 3x & x < 1 \\
b & x = 1 \\
x^3 - ax & x > 1 
\end{cases}$$

For what values of $a$ and $b$ is $f(x)$ continuous at $x = 1$? Justify your answer.

(b) (7 points) Define the function $g(x)$ by

$$g(x) = \begin{cases} 
x & x < 1 \\
2x - 1 & x \geq 1 
\end{cases}$$

Is $g(x)$ differentiable at $x = 1$? Justify your answer using the limit definition of derivative.
#3. The definition of a function $f(x)$ and a portion of its graph are shown below.

$$f(x) = \begin{cases} 
  x^2 & x < 0 \\
  0 & 0 \leq x < 2 \\
  x & 2 \leq x < 4 \\
  (x - 6)^2 & x \geq 4 
\end{cases}$$

(For (a), (b), and (c) you may state your answers without justification.)

(a) (5 points) Find all the values $x$ at which $f(x)$ is discontinuous.

(b) (5 points) Find all the values $x$ at which $f(x)$ is not differentiable.

(c) (4 points) Evaluate $\lim_{x \to 2^+} f(f(x))$. 

#4. The cost (in dollars) of producing $x$ cars is $C(x) = x^2 + 3x + 1$.

(a) (5 points) Find the average rate of change of $C$ with respect to $x$ when production is raised from $x = 4$ to $x = 5$.

(b) (5 points) Find the instantaneous rate of change of $C$ with respect to $x$ when $x = 5$. Justify your answer using limits.

(c) (4 points) Find the equation of the tangent line to the graph of $C$ at $x = 5$. 
#5. Let \( f(x) = 7x - 7 \).

(a) (8 points) Find the largest value of \( \delta \) such that if \( |x - 2| < \delta \), then \( |f(x) - 7| < \epsilon \). Express your answer in terms of \( \epsilon \).

(b) (6 points) If we require \( x \) to be in the interval \((2.5, 3.5)\), what is the smallest value of \( \epsilon \) such that \( |f(x) - 14| < \epsilon \)?
#6. (14 points) Let $g(x) = 3^{-x}$ and $h(x) = x$. Prove that there is some number $c$ such that $g(c) = h(c)$. Precisely state any theorems you use.
#7. (14 points) Use the limit definition of the derivative to find $f'(a)$ for

$$f(x) = \sqrt{x^2 + 1}.$$