Math 235 MINI-MIDTERM II Name: SOLUTIONS

Kusner 16 April 2013 ID#: EGO, SUPEREGO

(Use back for any calculations)

1) Suppose $V = \text{span}(\mathcal{B})$ is the vector space with basis $\mathcal{B} = \{e^{2t}, e^{3t}\}$.

(1) a) $\dim(V) = \boxed{2}$?

(2) b) The coordinates of $v = 7e^{2t} - \frac{1}{3}e^{3t}$ are $[v]_{\mathcal{B}} = \begin{bmatrix} \boxed{7} \\ \boxed{-1/3} \end{bmatrix}$?

(4) c) The matrix of the linear map $\frac{d}{dt} : V \to V$ is $\left[\frac{d}{dt}\right]_{\mathcal{B}\mathcal{B}} = \begin{bmatrix} \boxed{2} & \boxed{0} \\ \boxed{0} & \boxed{3} \end{bmatrix}$?

$$\left[\tfrac{d}{dt} e^{2t}\right]_{\mathcal{B}} \qquad \left[\tfrac{d}{dt} e^{3t}\right]_{\mathcal{B}}$$

2) Define an inner product $\langle p, q \rangle := p(0)q(0) + p(1)q(1)$ on the vector space $P_1 = \text{span}(\{1, t\})$ of degree-1 polynomials.

(1) a) $\| 1 \| = \boxed{\sqrt{2}}$?

(1) b) $\| t \| = \boxed{1}$?

(1) c) $\langle 1, t \rangle = \boxed{1}$?

(2) d) The angle between 1 and t is $\theta = \boxed{45^\circ = \frac{\pi}{4}}$? $\quad = \cos^{-1}\left(\frac{\langle 1, t \rangle}{\|1\| \|t\|}\right)$

(ε) extra credit

e) Find a unit vector $u \in P_1$ orthogonal to $v = t$ (Hint: apply Gram-Schmidt to the ordered basis $\{t, 1\}$.)

Let $w = 1 - \langle 1, t \rangle t$, which is $\perp$ to $v = t$.

Then $u = \frac{w}{\|w\|} = \frac{1-t}{\|1-t\|} = 1 - t$ since $\|1 - t\| = 1$.