Name (Last, First) ______________________ **ID #** __________

Signature ______________________________________

Instructor's Name ____________ **Section (01, 02, 03, etc.)** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **December 1st, 2020**
2:00-4:00 p.m. EST

Instructions

- Please turn off and put away all electronic devices. This is a closed book exam. No calculators, notes, or books are allowed.
- There are six (6) questions. If you are not printing out your exam booklet, please answer each question on a separate sheet of paper. Please write the question number and sub-question (if applicable), as well as your full name and student ID on top of each page of your solutions.
- After you have completed the exam, scan all pages of your solutions to a PDF file. You will have 30 minutes (4-4:30pm EST) after the exam is over to upload the PDF file to Gradescope. (Please make sure that you label/assign all pages appropriately after you upload them.) If, for any reason, you are unable to upload your solutions, please immediately email the PDF file containing your solutions to your instructor.
- Show all of your work, and be sure to organize it well. (**Answers given without proper justification may receive 0 credit.**)

QUESTION	PER CENT	SCORE
1	24	
2	15	
3	16	
4	15	
5	15	
6	15	
TOTAL	100	

#1. (24 points)

Parts (a), (b) and (c) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve one part, you should still attempt to answer the other parts.

(a) (8 points) Given a function $f(x) = \frac{1}{x}$. Use the definition of a derivative to find $f'(x)$.

(b) (8 points) Find the total distance traveled for a particle traveling in a horizontal motion from $t = 0$ to $t = 5$ seconds according to the position function: $s(t) = t^2 - 4t$. (Assume that the position given by the position function is measured in meters.)

(c) (8 points) Find the horizontal and vertical asymptotes of the function:

$$f(x) = \frac{x^2 + 3x + 1}{4x^2 - 9}.$$

#2. (15 points)

Parts (a), (b) and (c) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve one part, you should still attempt to answer the other parts.

Use LHospitals rule to evaluate the following limits:

(a) (5 points) $\displaystyle\lim_{x\to-\infty} \frac{x^2}{e^{1-x}}$.

(b) (5 points) $\displaystyle\lim_{x\to\infty} x \ln\left(1+\frac{3}{x}\right)$.

(c) (5 points) $\displaystyle\lim_{x\to\infty} \frac{x^2+e^{4x}}{2x-e^{x}}$.

#3. (16 points) Let $f(x) = x^3 - 12x^2 + 36x + 4$. We know that the domain of f is all real numbers and $f'(x) = 3x^2 - 24x + 36$, $f''(x) = 6x - 24$.

(a) (8 points) Find the interval(s) on which f is increasing. Find the interval(s) on which f is decreasing. Determine the x-coordinates of all local maxima and local minima.

(b) (8 points) Find the interval(s) where the function is concave up. Find the interval(s) where the function is concave down. Determine the x-coordinates of all points of inflection of the graph of f(x).

#4. (15 points)

Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (7 points) Among all the pairs of positive numbers that add up to 100, find the pair that will yield/have the largest product.

(b) (8 points) We have 40 m^2 of material to build a box with a square bottom and no top. Determine the dimensions of the box that will have the largest volume.

#5. (15 points)

Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (7 points) A particle is moving with acceleration $a(t) = 2t + 1$. Given that $s(0) = 4$ and $v(0) = -6$, find the function $s(t)$ describing the position of the particle.

(b) (8 points) An object was dropped (let go down) from a tower and hit the ground with a speed of 136 ft/s. What is the height of that tower? (Use $32 ft/s^2$ for the acceleration due to gravity.)

#6. (15 point)

Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (10 points) Given $g(x) = 1 + x - x^2$. Write a definite integral representing the exact area of the region under the curve $y = g(x)$ on the interval $[0, 1]$. Evaluate this integral using the definition of the definite integral as a limit of Riemann sums. You can use the following identities for sums of powers of consecutive positive integers.

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(b) (5 points) Express the integral $\int_2^5 \frac{x}{1+x^2}dx$ as a limit of Riemann sums. Do not evaluate the limit. (Use the right endpoints of the subintervals as the sample points.)