Name (Last, First) ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯ ID # ⎯⎯⎯⎯⎯⎯

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Instructor's Name ⎯⎯⎯⎯⎯⎯⎯⎯ Section (01, 02, 03, etc.) ⎯⎯⎯⎯⎯

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 Exam 2 November 4th, 2020
7:00-9:00 p.m. EST

Instructions

- Please turn off and put away all electronic devices. This is a closed book exam. No calculators, notes, or books are allowed.

- There are six (6) questions. Please write each question on one page of paper, mark the question number and sub-question number clearly, and write your full name & student ID number on top of each page of your solutions.

- After you have completed the exam, scan all pages of your solutions (one page per question) to a PDF file. You will have 30 minutes (9:00-9:30pm EST) after the exam is over to upload the PDF file to Gradescope. If, for any reason, you are unable to upload your solutions, please immediately email the PDF file containing your solutions to your instructor.

- Show all of your work, and be sure to organize it well. (**Answers given without proper justification may receive 0 credit.**)

QUESTION	PER CENT	SCORE
1	18	
2	16	
3	16	
4	16	
5	16	
6	16	
Free	2	
TOTAL	100	

#1. (18 points) Find the derivatives of the following functions. You do NOT need to simplify your answer.

(a) (6 points) $f(x) = \cos\left(\dfrac{1 - e^{-x}}{1 + e^{-x}}\right)$.

(b) (6 points) $g(x) = e^x\sqrt{x} + \ln(\tan(x))$

(c) (6 points) $h(x) = \arcsin(2x) + \sec(\sqrt{x})$.

#2. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) Given that $y = x^{3\cos(x)}$, find $\dfrac{dy}{dx}$.

(b) (8 points) Given that $e^y \cos(x) = 100 + \sin(xy^2)$, find $\dfrac{dy}{dx}$.

#3. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 3$ where the tangent line is horizontal.

(b) (8 points) If a ball is thrown vertically upward with a velocity of $80\,ft/s$, then its height after t seconds is $s = 80t - 16t^2$. What is the maximum height reached by the ball?

#4. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 320. Find the number of bacteria after 2 hours. (Your answer may involve logarithms and the number e.)

(b) (8 points) A cylindrical tank with radius 8 m is being filled with water at a rate of $4m^3/min$. How fast is the height of the water increasing?

#5. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) Use a **linear approximation (or a differential)** to (linearly) estimate (or approximate) $e^{0.02}$.

(b) (8 points) Compute Δy and dy of function $y(x) = \sqrt{x}$ for the given values of $x = 1$ and $dx = \Delta x = 0.1$.

#6. (16 point) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 6x^2 + 9x + 10$ on the given interval [-1, 2].

(b) (8 points) Suppose that $2 \leq f'(x) \leq 3$ for all values of x. What are the minimum and maximum possible values of $f(7) - f(2)$?