**Name (Last, First)** ______________________ **ID #** __________

**Signature** ______________________________________

**Instructor** __________________ **Section (01, 02, 03, etc.)** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

**Math 131** **Final Exam** **December 16, 2019**
**1:00-3:00 p.m.**

**Instructions**

- Please turn off and put away all electronic devices.
- There are seven (7) questions.
- Do all of your work in this exam booklet. You may continue work on the backs of pages and the blank page at the end, but if you do so, indicate where.
- No calculators, notes, or books are allowed.
- Show all of your work, and be sure to organize it well. (Answers given without supporting work may receive 0 credit.) Please **circle your final answers.**
- Be ready to show your UMass ID card when you hand in your exam booklet.
- **Good Luck!**

| QUESTION | POINTS | SCORE |
|---|---|---|
| 1 | 16 | |
| 2 | 14 | |
| 3 | 16 | |
| 4 | 12 | |
| 5 | 14 | |
| 6 | 14 | |
| 7 | 14 | |
| TOTAL | 100 | |

#1. For each question (I)–(IV), please **circle** all correct responses.

(I) (4 points) For what value(s) of $a$ is the function $g(x)$ continuous at $x = 3$?

$$g(x) = \begin{cases} 2x - a & \text{if } x < 3 \\ x + 2a & \text{if } x \geq 3 \end{cases}$$

[A] $a = 0$ [B] $a = -1$ [C] $a = 2$ [D] $a = 1$

(II) (4 points) The definition of the first derivative of a function $f(x)$ is

[A] $f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ [B] $f'(x) = \lim_{h \to 0} \frac{f(a+h) + f(a)}{h}$

[C] $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ [D] $f'(x) = \lim_{h \to 0} \frac{f(x+h) + f(x)}{h}$

(III) (4 points) What is the first derivative of the function $y = x^x$?

[A] $y' = x^x(\ln x + x)$ [B] $y' = x^x(\ln x + 1)$

[C] $y' = x^x(\ln x + x^x)$ [D] $y' = x(\ln x + 1)$

(IV) (4 points) Consider the function $f(x) = x^2 - 2x + 3$ on the interval $[0, 3]$. It is true that...

[A] ...There is a critical number of $f(x)$ in $(0, 3)$.

[B] ...There are two critical numbers of $f(x)$ in $(0, 3)$.

[C] ...The function $f(x)$ attains its absolute maximum value at $x = 0$.

[D] ...The function $f(x)$ attains its absolute minimum value at $x = 1$.

#2. Use L'Hospital's rule to evaluate the following limits.

(a) (3 points) $\lim_{x\to 0} \frac{\sin(2x)}{\tan(7x)}$

(b) (5 points) $\lim_{x\to\infty} \frac{(\ln x)^2}{x}$

(c) (6 points) $\lim_{x\to\infty} x \sin\left(\frac{\pi}{x}\right)$

#3. Multi-part question.

Let $f(x) = \dfrac{x^3}{3} - \dfrac{x^2}{2}$. We know that the domain of $f$ is all real numbers and $f'(x) = x(x-1), f''(x) = 2x - 1$.

(a) (8 points) Find the **interval(s)** on which $f$ is increasing. Find the **interval(s)** on which $f$ is decreasing. Determine the $x$**-coordinates** of all local maxima and local minima.

(b) (8 points) Find the **interval(s)** where the function is concave up. Find the **interval(s)** where the function is concave down. Determine the $x$**-coordinates** of all points of inflection of the graph of $f(x)$.

#4. (12 points)

A rectangle has area 225 square meters. Determine the dimensions that minimize the perimeter, and find the minimum possible perimeter.

#5. (14 points)

A remote-control toy car is traveling at 50 cm/s when the brakes are fully applied, producing a constant deceleration of 10 cm/s$^2$. What is the distance covered before the car comes to a stop?

#6. Multi-part question.

Consider the function $f(x) = x$.

(a) (6 points) Use a right-endpoint Riemann sum with $n = 3$ rectangles to estimate the area of the region under the curve $y = f(x)$ on the interval $[0, 3]$.

(b) (8 points) Write a definite integral representing the exact area of the region under the curve $y = f(x)$ on the interval $[0, 3]$. Then evaluate this integral using the definition of the definite integral as a limit of Riemann sums. You can use the following identity for the sum of powers of consecutive natural numbers.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

#7. Part (a) and part (b) are independent.

(a) (6 points) Suppose that $\int_0^8 f(x)\ dx = 10$ and $\int_0^8 g(x)\ dx = 5$.

Find $\int_0^8 [4f(x) + 5g(x)]\ dx$.

(b) (8 points) Evaluate the integral $\int_0^2 \sqrt{4 - x^2}\ dx$ by interpreting it in terms of areas.

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