

Name (Last, First) \_\_\_\_\_ ID # \_\_\_\_\_

Signature \_\_\_\_\_

Instructor \_\_\_\_\_ Section (01, 02, 03, etc.) \_\_\_\_\_

UNIVERSITY OF MASSACHUSETTS AMHERST  
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131

Midterm Exam 1

October 2, 2019  
7:00-9:00 p.m.

**Instructions**

- Please turn off and put away all electronic devices.
- There are seven (7) questions.
- Do all of your work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so, indicate where.
- No calculators, notes, or books are allowed.
- Show all of your work, and be sure to organize it well. (Answers given without supporting work may receive 0 credit.)
- Be ready to show your UMass ID card when you hand in your exam booklet.
- **Good Luck!**

QUESTION	POINTS	SCORE
1	14	
2	14	
3	14	
4	14	
5	14	
6	14	
7	14	
Free	2	
TOTAL	100	

#1. Shenanigans with Limits

(a) (7 points) Determine whether the function  $f(x) = \frac{3x^2 + 10x + 8}{x + 2}$  has any horizontal or vertical asymptotes. Justify your answers using limits.

(b) (7 points) If the following limit exists, then evaluate it. If the limit does not exist, then explain why.

$$\lim_{x \rightarrow 3} \frac{x^2 + 5x - 6}{x - 3}$$

#2. Continuous and Differentiable Functions

(a) (7 points) Consider the following function:

$$g(x) = \begin{cases} e^x & \text{if } x < 3 \\ x + 2a & \text{if } x \geq 3 \end{cases}$$

Use the definition of a continuous function to determine the value of  $a$  so that  $g(x)$  is continuous at  $x = 3$ .

(b) (7 points) Consider the following function:

$$k(x) = \begin{cases} 2x + 1 & \text{if } x \leq 4 \\ x + 5 & \text{if } x > 4 \end{cases}$$

Use the definition of a differentiable function to determine whether or not  $h(x)$  is differentiable at  $x = 4$ .

### #3. Calculating Limits

For each of the following limits, either show that the limit does not exist or evaluate the limit. As always, show all of your work and fully simplify your results.

(a) (5 points)  $\lim_{x \rightarrow 1} \frac{x^2 - x - 2}{3x^2 + x - 2}$

(b) (5 points)  $\lim_{u \rightarrow -1} \cos \frac{\pi}{3u}$

(c) (4 points)  $\lim_{t \rightarrow \infty} \frac{t^2 + 4}{-5t - 5}$

#4. Rates of Change

On the planet Kerbal, Jebediah throws a ball into the air. The height of the ball is given by  $y(t) = -2t^2 + 16t$  meters above sea level, where  $t$  is measured in seconds.

(a) (4 points) What is the ball's average velocity for the time interval  $1 \leq t \leq 2$ ?

(b) (5 points) Let  $h$  be an unknown constant. What is the ball's average velocity for the time interval  $1 \leq t \leq 1 + h$ ?

(c) (5 points) What is the ball's instantaneous velocity when  $t = 1$ ? Justify using limits.

#5. (14 points) Definition of a Limit

Use the  $\varepsilon - \delta$  definition of a limit to prove that

$$\lim_{x \rightarrow -3} (1 - 4x) = 13$$

#6. (14 points) A Fun Application

Show that the polynomial  $p(x) = x^5 + 2x^2 + x - 10$  has a real root. You do not need to find the root, but must carefully justify its existence.

#7. (14 points) Definition of the Derivative

Let  $f(x) = \frac{3}{x}$ , where  $x > 0$ . Use the limit definition of the derivative to find  $\frac{df}{dx}$ .



*This page intentionally left blank*