Name (Last, First) ____________________ **ID #** __________

Signature ______________________________________

Lecturer ________________ **Section (01, 02, 03, etc.)** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 233 Exam 1 February 21, 2018 7:00-9:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are six (6) questions and eight (8) pages including this one and the blank page at the end.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- In problems that require reasoning or algebraic calculation, it is not sufficient just to write answers only. You must explain how you arrived at your answers, and show your algebraic calculations.
- You can leave answers in terms of fractions and square roots except where rounded decimal values are specified.
- **Calculators, crib sheets, notes, and textbooks are not allowed.**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	POINTS	SCORE
1	20	
2	15	
3	15	
4	15	
5	20	
6	15	
TOTAL	100	

#1. *Short answers: 4 points each, no partial credit. Write answers in boxes.*

(a) Find the center of the sphere $x^2 + y^2 + z^2 - 12x + 4y - 6z + 45 = 0$.

The center is $\boxed{}$.

(b) Find the distance from the point $(2, -7, 4)$ to the x-axis.

The distance is $\boxed{}$.

(c) Find the work done by the force vector $\mathbf{F} = \langle 3, -4, 2 \rangle$ in moving an object along the line segment from point $P = (0, 1, 2)$ to point $Q = (4, -7, 3)$. Ignore units of measure.

$W = \boxed{}$.

(d) Find the scalar projection of $\mathbf{v} = (-2/3)\,\mathbf{i} + 8\mathbf{j}$ onto the vector $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$.

$\text{comp}_{\mathbf{u}}\,\mathbf{v} = \boxed{}$.

(e) Let $\mathbf{r}(t) = \left\langle 3t^2, 2t + 1, \frac{3}{t} \right\rangle$. Find $\|\mathbf{r}'(1)\|$.

$\|\mathbf{r}'(1)\| = \boxed{}$.

#2. *(15 points)* The points $A(0, 0, 0)$, $B(1, 1, 1)$, $C(1, 5, 1)$, and $D(0, 4, 0)$ form the four vertices of the parallelogram $ABCD$.

(a) What is the length of the longer of the two diagonals?

(b) Find the area of the parallelogram. *(Hint: use sides AB and AD)*

(c) Find an equation of the plane containing the parallelogram.

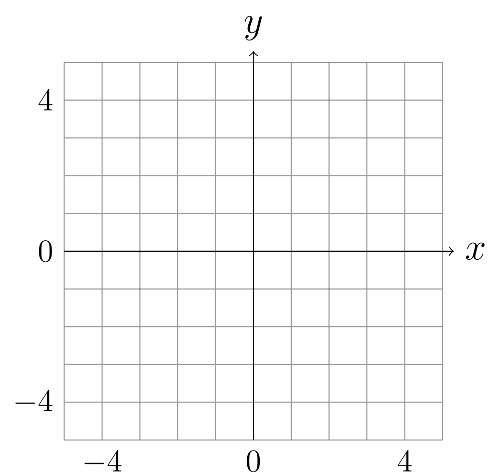
#3. *(15 points)* Let $A = (3, 5, 2)$ and let $B = (7, 9, 4)$.

(a) Find the coordinates of the point at which the line containing both points A and B intersects the xy-plane.

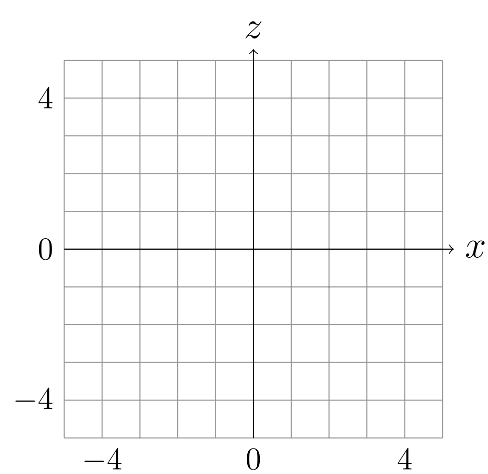
(b) Find a vector function $\mathbf{r}(t)$ that describes the **line segment** from point A to point B.

#4. *(15 points)* Let S be a surface in $\mathbb{R}^3$ described by the equation $x^2 + 4y^2 - z = 0$.

(a) Sketch the trace of S given the plane $z = 16$.



(b) Sketch the trace of S given the plane $y = 2$.



(c) Find parametric equations for the curve of intersection of the cylinder $x^2 + y^2 = 1$ with the surface S.

#5. *(20 points)* A particle moves along a path in space described by $\mathbf{r}(t) = \langle 4\sin t, 3t, -4\cos t\rangle$.

(a) Find parametric equations for the tangent line to the path where $t = 0$.

(b) Find the length the path from $t = 1$ to $t = 4$.

#6. *(15 points)* A moving particle starts at an initial position $\mathbf{r}(0) = \langle 1, 3, 1 \rangle$ and with initial velocity $\mathbf{v}(0) = \langle 2, 9, 3 \rangle$. Its acceleration vector function is $\mathbf{a}(t) = \langle 2, 0, 12t \rangle$.

(a) Find the velocity vector function $\mathbf{v}(t)$.

(b) Find the position vector function $\mathbf{r}(t)$.

(c) Set up but do not evaluate an integral that gives the distance traveled by the particle from $t = 1$ to $t = 2$.

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