DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 233 **<u>EXAM 2</u>** **Fall 2018**

NAME:________________________________ Student ID:____________________

Section Number:____________________ Instructor's Name: ____________________

In this exam there are NINE PAGES, including this one, and SEVEN problems. Make sure you have them all. You must SHOW ALL WORK. Calculators, textbooks, class notes, formula sheets, etc. are NOT allowed. BOX your final answers.

1.	(14)	__________
2.	(14)	__________
3.	(14)	__________
4.	(14)	__________
5.	(14)	__________
6.	(15)	__________
7.	(15)	__________
Total	(100)	__________

1. A particle moves along the curve

$$\mathbf{r}(t) = (t^3/3, t^2, 2t).$$

Find the length of the path traveled by the particle between $t = 1$ and $t = 3$.

2. Find the position vector function of a particle that has an acceleration function

$$\mathbf{a}(t) = e^{t/2}\mathbf{i} + \mathbf{k},$$

an initial velocity $\mathbf{v}(0) = 2\mathbf{j}$, and an initial position $\mathbf{r}(0) = 0$.

3. Consider the function

$$f(x,y) = \frac{x^3 y}{x^6 + y^2}.$$

(a) Find the limit of $f(x,y)$ as (x,y) approaches the origin *along a straight line* of slope m, where m is a real number.

(b) Find the limit of $f(x,y)$ as (x,y) approaches the origin *along the curve* $y = x^3$.

(c) Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Explain why or why not.

4. (a) Find the linear approximation to $f(x, y) = \sqrt{xy + 1}$ at the point $(4, 6)$, and use this to estimate $f(3.9, 5.9)$.

(b) The volume of a square pyramid is measured as 270 cubic centimeters with a possible error of ± 3 cubic centimeters. Its height is measured as 10 centimeters, with a possible error of ± 0.1 centimeters. Use differentials to estimate the maximum error in calculating the side length of the square base from the measured volume and height. You must include units in your final answer. (Recall that the volume of a pyramid with a square base of side length l and height h is $V = \frac{1}{3} l^2 h$.)

5. (a) Find the directional derivative of the function $f(x,y) = \ln(x^2+y^2)$ in the direction of $\mathbf{v} = \langle 3, -2 \rangle$ at the point $P = (1,0)$.

(b) Find the maximum possible directional derivative of $f(x,y) = \ln(x^2+y^2)$ at the point $P = (1,0)$ and the unit vector in the direction in which it occurs.

6. Find all local maxima, minima, and saddle points of the function

$$f(x, y) = x^3 - 3x + x^2y^2.$$

Be sure to specify the type of each point you find.

7. Use the method of Lagrange multipliers to find the absolute maximum and absolute minimum values of the function $f(x, y) = xe^y$ on the curve given by $x^2 + y^2 = 2$.

Scratch paper