Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Lecturer __________________________ **Section (01, 02, 03, etc.)** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 233 **Final Exam** **December 17, 2019**
8:00-10:00 a.m.

Instructions

- **Turn off all cell phones!** Put away all electronic devices such as iPods, iPads, laptops, etc.
- There are six (6) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate clearly where your work is for the grader.
- Calculators are **not** allowed, nor are formula sheets or any other external materials.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Unless indicated otherwise, you must show work to obtain credit for your answers.**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
TOTAL	120	

1. (20 points) For each question, please select the best response. Please clearly indicate your choice; ambiguous answers will not receive credit. In this problem, there is *no partial credit* awarded and it is *not necessary to show your work*.

(a) (4 points) Suppose f is a differentiable function of x and y, and $g(r,t) = f(2r - t, r^2 - 2t)$. Use the following table of values to calculate $g_t(0,0)$.

(x, y)	$(-1, -2)$	$(-1, 0)$	$(0, 0)$	$(1, 0)$
f	1	2	3	4
f_x	A	B	C	D
f_y	E	F	G	H

(i) $A + 4$ (ii) $-C - 2G$ (iii) $-B + 2F$
(iv) $2 - E$ (v) $2 - G$ (vi) $2B - F$

(b) (4 points) Compute the following limit:

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - 2y^2}{x^2 + 2y^2}.$$

(i) -2 (ii) -1 (iii) 0

(iv) 1 (v) 2 (vi) The limit does not exist.

(c) (4 points) Let $\vec{r}(t) = \langle 2t, t^2 - 1, 2t + 3\rangle$. Then the tangent line at the point $(4, 3, 7)$ has the equation

(i) $\langle 2t, 2t - 1, 2t + 3\rangle$ (ii) $\langle t + 4, t - 1, 2t + 3\rangle$ (iii) $\langle 2t, 4t - 1, 2t + 3\rangle$

(iv) $\langle 2t + 2, 2t + 3, 2t + 7\rangle$ (v) $\langle 2t + 4, 4t + 3, 2t + 7\rangle$ (vi) $\langle t + 4, 2t + 3, t + 7\rangle$

(d) (4 points) Use differentials to estimate the error in cubic inches of computing the volume of a cylindrical can with height $H = 10$ inches and radius $R = 6$ inches, and where each measurement has an error of at most 0.5 inches. (Hint: the volume is $V = \pi R^2 H$.)

(i) 48π (ii) 60π (iii) 72π

(iv) 78π (v) 120π (vi) 156π

(e) (4 points) Let $\vec{F}(x, y, z)$ be a continuously differentiable 3-dimensional vector field. Which of the following expressions makes sense and gives a scalar function?

(i) $\operatorname{div}(\operatorname{curl} \vec{F})$ (ii) $\operatorname{curl}(\operatorname{div} \vec{F})$ (iii) $\operatorname{curl}(\nabla \vec{F})$

(iv) $\operatorname{div}(\operatorname{div} \vec{F})$ (v) $\nabla(\nabla \vec{F})$ (vi) $\nabla(\operatorname{curl} \vec{F})$

2. (20 points) Let $f(x, y) = x^3 + xy^2 + 6x^2 + y^2$.

(a) (5 points) Compute the gradient of f.

(b) (15 points) Find all critical points of f and classify them as local maxima, local minima, or saddle points.

3. (20 points) Let $\vec{\boldsymbol{F}}(x, y)$ be the vector field in the plane given by $\langle x^2+y^2+y, 2xy\rangle$.

(a) (5 points) Is $\vec{\boldsymbol{F}}$ conservative? Why or why not?

(b) (15 points) Evaluate $\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$, where C is the path beginning at the origin and going to $(1, 2)$ along a line segment.

4. (20 points) Let C be the boundary of the quarter circle with radius 1, oriented counterclockwise (Figure 1). Evaluate $\oint_C (e^x + y^2)\,dx + (e^y + x^2)\,dy$.

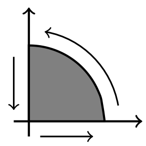

Figure 1

5. (20 points) Let C be the closed path consisting of line segments that move in order from the origin to $(1,0,0)$, then to $(1,1,1)$, then to $(0,1,1)$, and then back to the origin. A particle is moved along C under the influence of the force field $\vec{\boldsymbol{F}}(x,y,z) = z^2\vec{\mathbf{i}} + xy\vec{\mathbf{j}} + y^2\vec{\boldsymbol{k}}$. Compute the work done $\oint_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$.

6. (20 points) Let S be the surface of the cube bounded by the planes $x = 1$, $y = 1$, and $z = 1$ and the coordinate planes (that is, S is the surface of the cube with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, $(1, 1, 0)$, $(1, 0, 1)$, $(0, 1, 1)$, and $(1, 1, 1)$). Assume that S is oriented with *outward* pointing normal vector. Let $\vec{\boldsymbol{F}}(x, y, z)$ be the vector field $\langle x^2 + \sin z, xz, z^2 + e^x\rangle$. Compute the flux of $\vec{\boldsymbol{F}}$ across S.

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