Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Lecturer _________________________ **Section (01, 02, 03, etc.)** _________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 233 **Exam 1** **October 15, 2018**
7:00-9:00 p.m.

Instructions

- **Turn off all cell phones!** Put away all electronic devices such as iPods, iPads, laptops, etc.
- There are five (5) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate clearly where your work is for the grader.
- Calculators are **not** allowed, nor are formula sheets or any other external materials.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Unless indicated otherwise, you must show work to obtain credit for your answers.**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

1. (20 points) For each question, please select the best response. Please clearly indicate your choice; ambiguous answers will not receive credit. In this problem, there is *no partial credit* awarded and it is *not necessary to show your work.*

(a) (4 points) Find the area of the triangle with vertices $(2,1,1)$, $(1,2,1)$, $(1,1,2)$.

(i)	1	(ii)	2	(iii)	$\sqrt{3}$
(iv)	$\sqrt{3}/2$	(v)	$\sqrt{3}/4$	(vi)	$\sqrt{6}/2$

(b) (4 points) Find the **cosine** of the angle between the two planes $x + 2y = 0$ and $x + 2z = 3$.

(i)	$2/3$	(ii)	$3/4$	(iii)	$\sqrt{3}/2$
(iv)	$1/5$	(v)	$1/\sqrt{2}$	(vi)	$1/2$

(c) (4 points) Find the **unit tangent vector** to the parametric curve $\vec{r}(t) = \langle \sin t, 2t, t^2 \rangle$ at $t = 0$.

(i)	$\langle 1/\sqrt{5}, 2/\sqrt{5}, 0 \rangle$	(ii)	$\langle 1/\sqrt{5}, -2/\sqrt{5}, 0 \rangle$	(iii)	$\langle -2/\sqrt{5}, 1/\sqrt{5}, 0 \rangle$
(iv)	$\langle 2/\sqrt{5}, 1/\sqrt{5}, 0 \rangle$	(v)	$\langle -1/\sqrt{5}, -2/\sqrt{5}, 0 \rangle$	(vi)	$\langle 2/\sqrt{5}, -1/\sqrt{5}, 0 \rangle$

(d) (4 points) Describe the **level curves** of the function $f(x, y) = \sqrt{1 - x^2 - 2y^2}$.

(i)	concentric circles	(ii)	concentric ellipses (not circles)
(iii)	parabolas with the same vertex	(iv)	parabolas with different vertices
(v)	hyperbolas with the same vertex	(vi)	hyperbolas with different vertices

(e) (4 points) The function $f(x, y) = x^2 + y^2 + 3xy$ has one critical point. Determine its location and type.

(i)	$(0,0)$, saddle point	(ii)	$(0,0)$, maximum point
(iii)	$(0,0)$, minimum point	(iv)	$(2,1)$, saddle point
(v)	$(2,1)$, maximum point	(vi)	$(2,1)$, minimum point

2. (20 points)

(a) (6 points) Let P be the plane through the points $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$. Find an equation for P.

(b) (6 points) Let L be the line through the origin in the direction of $\vec{\boldsymbol{r}} = \langle 2,-2,-3\rangle$. Find parametric equations for L.

(c) (8 points) Does L intersect P? If yes, find the point of intersection. If not, find the distance between L and P.

3. (20 points) Suppose a particle moves with position function $\vec{\boldsymbol{r}}(t) = \langle t^2, t\sqrt{2}, (\ln t)/2\rangle$, where $t > 0$.

(a) (8 points) Find the velocity and acceleration of the particle.

(b) (12 points) Find the distance traveled by the particle from $t = 1$ to $t = e$.

4. (20 points) Let $f(x, y) = 2x^2y + x + y^3$.

(a) (6 points) Compute ∇f.

(b) (7 points) Compute the directional derivative of f at the point $(1, 2)$ in the direction that forms an angle (measured clockwise) of $\pi/3$ with the x-axis.

(c) (7 points) Find an equation of the tangent plane to the graph of f at the point $(1, 2, f(1, 2))$.

5. (20 points) Let $f(x, y) = x^2 - 4x + y^2 - 2y + 2$. Find the absolute maximum and the absolute minimum of f on the closed rectangle with vertices $(0, 0)$, $(4, 0)$, $(0, 4)$, $(4, 4)$.

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