DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 233 **MAKEUP EXAM 2** **Fall 2014**

NAME:____________________________________

Section Number:_________________________ Instructor's Name: _________________________

In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You **must explain** how you arrived at your answers, and show your algebraic calculations.

You can leave answers in terms of fractions and square roots. It is not necessary to algebraically simplify partial derivatives.

1.	(25)	
2.	(15)	
3.	(15)	
4.	(15)	
5.	(10)	
6.	(10)	
7.	(10)	
Total		

Perfect Paper ⟶ 100 Points.

There are 9 pages (or sides, including this one) in this exam and seven problems. Make sure you have them all before you begin! There is an additional blank page at the end of the exam if you need more space to write down your solutions.

1. (25 points) For the function $f(x, y) = x^2y - y^5$, calculate:

   (a) (2 points) $f_x(x, y) =$

   (b) (2 points) $f_y(x, y) =$

   (c) (2 points) $f_{yx}(x, y) =$

   (d) (3 points) What is the gradient $\nabla f(x, y)$ of f at the point $(3, 1)$?

   $\nabla f(3, 1) =$

   (e) (5 points) Calculate the directional derivative of $f(x, y)$ at the point $(3, 1)$ in the direction of the vector $\mathbf{v} = \langle -3, 4 \rangle$?

   (f) (3 points) What is the maximum rate of change of $f(x, y)$ at the point $(3, 1)$?

   (g) (5 points) What is the linearization $L(x, y)$ of $f(x, y)$ at $(3, 1)$?

   (h) (3 points) Use the linearization $L(x, y)$ in the previous part to estimate $f(3.1, 0.9)$.

2. (15 points) A hiker is walking on a mountain path. The surface of the mountain is modeled by $z = 100 + 8x - 4x^2 - 5y^2$. The positive x-axis points to **East** direction $\langle 1, 0 \rangle$, the negative x-axis points **West** $\langle -1, 0 \rangle$, the positive y-axis points **North** $\langle 0, 1 \rangle$ and the negative y-axis points **South** $\langle 0, -1 \rangle$. Justify your answers.

   (a) (5 points) Suppose the hiker is now at the point $P(20, 10, 133)$ heading **East**, is she **ascending** or **descending**? Explain your answer.

   (b) (5 points) When the hiker is at the point $Q(2, 1, 250)$, in which direction on her map should she initially head to **descend** most rapidly?

   (c) (5 points) When the hiker is at the point $Q(2, 1, 250)$, in which two directions on her map can she initially head to **neither** ascend nor descend (to keep traveling at the same height)?

3. a) (5 points) Let $f(x, y)$ be a differentiable function with the following particular **partial derivatives** for $f_x(x, y)$ and $f_y(x, y)$ at certain points (x, y):

$$f_x(0,1) = 5, \quad f_y(0,1) = -3, \quad f_x(-2,1) = 1, \quad f_y(-2,1) = 4$$

(You are given more values than you will need for this problem.) Suppose that x and y are functions of variable t: $x = t^2 - 3t$; $y = t^4$, so that we may regard f as a function of t. Compute the derivative of f with respect to t when $t = 1$.

b) (10 points) Use the Chain Rule to find $\frac{\partial z}{\partial s}$ when $s = 1$ and $t = 2$, where

$$z = 3xy + x^2y - 2y^3; \quad x = \log(t + 2s) \text{ and } y = 3t + s^2.$$

4. Consider the surface defined by the function $x^2 - 3y^2 + xy + z^3 + z = 6$ and the point $P(2, 0, 1)$ which lies on the surface.

   (a) (10 points) Find an equation of the tangent plane to the surface at P. (Hint: Use properties of the gradient to find a normal vector to the surface at the point $P(2, 0, 1)$.)

   (b) (5 points) Find parametric equations of the normal line to the surface at P.

5. (15 points) Let $f(x, y) = -2xy^2 + x^2 + 6x + 12y^2$

   (a) (5 points) Find all critical points of f.

   (b) (5 points) Classify the critical points of f as local maxima, local minima or saddle points.

6. (10 points) A flat circular plate has the shape of the region $2x^2 + 6y^2 \leq 24$. The plate (including the boundary $2x^2 + 6y^2 = 24$) is heated so that the temperature at any point (x, y) on the plate is given by $T(x, y) = 3x^2 + 2y^2 + 8y$. Find the temperatures at the **hottest** and the **coldest points** on the plate, including the boundary $2x^2 + 6y^2 = 24$.

7. Suppose a particle moving in space has velocity

$$v(t) = \langle \cos(3t), \sin t, 6e^{t+1} \rangle$$

and initial position $r(0) = \langle 3, 0, 1 \rangle$.

(a) (5 points) Find the position vector function $r(t)$.

(b) (5 points) Write down an integral formula for the distance **D** traveled from time $t = 1$ to time $t = 5$ but do **not** evaluate this integral.