DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 233 **Makeup EXAM 2** **Fall 2017**

YOUR NAME: ____________________________________

Check the box next to your section:

☐ 1, Andrew Havens, 9:05 - 9:55, MWF	☐ 7, Luca Schaffler, 1:00 - 2:15, TuTh
☐ 2, Maria Nikolaou, 11:15 - 12:05, MWF	☐ 8, Luca Schaffler, 2:30 - 3:45, TuTh
☐ 3, Noriyuki Hamada, 12:20 - 1:10, MWF	☐ 9, William Meeks, 8:30 - 9:45, TuTh
☐ 4, Noriyuki Hamada, 1:25 - 2:15, MWF	☐ 10, Sean Hart 2:30 - 3:45, MW
☐ 5, Dinakar Muthiah, 10:00 - 11:15, TuTh	☐ 11, Maria Nikolaou 10:10 - 11:00, MWF
☐ 6, Dinakar Muthiah, 11:30 - 12:45, TuTh	☐

In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You must explain how you arrived at your answers, and show your algebraic calculations.

You can **leave answers in terms of fractions and square roots**.

$< x, y, z >$, $[x, y, z]$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$; are all permissible notations for the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

1.	(30)	________
2.	(15)	________
3.	(15)	________
4.	(10)	________
5.	(10)	________
6.	(10)	________
7.	(10)	________
Total		________

Perfect Paper $\longrightarrow$ 100 Points.

There are 9 pages, including this one and a blank one at the end, in this exam and 7 problems. Make sure you have them all before you begin!

1. For the function $f(x, y) = x^2 + 2xy + y^4$, calculate:

   (a) (2 points) $f_x(x, y) =$

   (b) (2 points) $f_y(x, y) =$

   (c) (2 points) $f_{xy}(x, y) =$

   (d) (3 points) What is the gradient $\nabla f(x, y)$ of f at the point $(2, 1)$?

   $\nabla f(2, 1) =$

   (e) i. (2 point) Calculate the unit vector $\mathbf{u}$ in the direction of the vector $\mathbf{v} = \langle -3, 4 \rangle$.

   ii. (4 points) Calculate the directional derivative of $f(x, y)$ at the point $(2, 1)$ in the direction of the vector $\mathbf{v} = \langle -3, 4 \rangle$.

   (f) (5 points) What is the maximum rate of change of $f(x, y)$ at the point $(2, 1)$?

   (g) (5 points) What is the linearization $L(x, y)$ of $f(x, y)$ at $(2, 1)$?

   (h) (5 points) Use the linearization $L(x, y)$ in the previous part to estimate $f(1.9, 1.1)$.

2. Consider the surface defined by $x^3 + y^2 + yz + z^2 = 6$ and let $P = (-1, 1, 2)$ which lies on the surface. (Hint: Consider the surface as a level set of the function $F(x, y, z) = x^3 + y^2 + yz + z^2$.)

   (a) (10 points) Find an equation of the tangent plane to the surface at the point P.

   (b) (5 points) Find parametric equations of the normal line to the surface at P.

3. A hiker is walking on a mountain path. The surface of the mountain is modeled by the surface $z = 90 - x - 7x^2 + 2y^2$. The positive x-axis points to **East** direction $\langle 1, 0\rangle$, the negative x-axis points **West** $\langle -1, 0\rangle$, the positive y-axis points **North** $\langle 0, 1\rangle$ and the negative y-axis points **South** $\langle 0, -1\rangle$. Justify your answers.

   (a) (5 points) Suppose the hiker is now at the point $P(3, -1, 26)$ heading **West**, is she **ascending** or **descending**? Explain your answer.

   (b) (5 points) When the hiker is at the point $Q(2, 0, 60)$, in which direction on her map should she initially head to **descend** most rapidly?

   (c) (5 points) When the hiker is at the point $Q(2, 0, 60)$, in which two directions on her map can she initially head to **neither** ascend nor descend (to keep traveling at the same height)?

4. (a) (5 points) The pressure P (in kilopascals), volume V (in liters), and temperature T (in degrees kelvin) of a mole of an ideal gas are related by the equation

$$P(T, V) = \frac{8.31T}{V}$$

Find the rate at which the pressure is changing when the temperature is 400 K and increasing at a rate of 0.3 K/s and volume is 100 L and increasing at a rate of 0.1 L/s. You can consider P, V, T to be functions of time t and then apply the chain rule to calculate $P'(t) = \frac{dP}{dt}$. Please include the physical units in your final answer or 1 point will be taken off.

(b) (5 points) Use the chain rule to find $\frac{\partial z}{\partial s}$ when $s = 4$ and $t = 3$, where

$$z = 2x^3 - xy + y - y^2; \quad x = s^2 - t \text{ and } y = 2e^{(s - \frac{4}{3}t)}.$$

5. Let $f(x, y) = 2x^2 - xy^2 + y^2$.

   (a) (5 points) Find all critical points of f.

   (b) (5 points) Classify the critical points of f as local maxima, local minima or saddle points.

6. (10 points) A flat elliptical plate has the shape of the region $6x^2 + 2y^2 \leq 32$. The plate is heated so that the temperature in degrees Celsius at any point (x, y) on the plate is given by $T(x, y) = 3x^2 + 2y^2 - 4y$. Find the temperatures at the **hottest** and the **coldest points** on the plate, including the boundary $6x^2 + 2y^2 = 32$. Please include the physical units in your final answer or 1 point total will be taken off. (Hint: You need to look for maximum and minimum values in both the interior of the ellipse as well as the boundary ellipse.)

7. A baseball is hit when it is 1 m above the ground. It leaves the bat with an initial speed of 60 m/sec, making an angle of 45° with the horizontal, which is in the direction of the unit vector $\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$ when we consider the problem in the (x, y)-plane where y represents the height of the ball with initial position $(0, 1)$. Assume that air resistance is negligible and the only external force is due to gravity. You do NOT need to include the physical units in your final answers.

   (a) (5 points) Find a vector equation for the velocity $\mathbf{v}(t)$ of the ball at time t, based on the acceleration due to gravity, $\mathbf{a} = -g\,\mathbf{j}$, with gravitational constant $g = 10$ m/sec^2; here $\mathbf{j} = \langle 0, 1 \rangle$ is the upward vertical direction.

   (b) (5 points) Find a vector equation for the position $\mathbf{r}(t)$ of the ball at time t, based on the acceleration due to gravity, $\mathbf{a} = -g\,\mathbf{j}$, with gravitational constant $g = 10$ m/sec^2; here $\mathbf{j} = \langle 0, 1 \rangle$ is the upward vertical direction. To find $\mathbf{r}(t)$ you may use your formula for $\mathbf{v}(t)$ in the previous part 7(a).

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