

MATH 233 PRACTICE MT#2, VERSION #2

DISCLAIMER: This practice exam is intended to give you an *idea* about what a two-hour midterm is like. It is not possible for any one exam to cover every topic, and the *content, coverage and format of your actual exam could be different from this practice exam.*

PART I: MULTIPLE CHOICE PROBLEMS. You only need to give the answer; no justification is needed.

#I-1. What is the geometric object defined by the **spherical coordinates equation** $\phi = \pi/3$?

- (a) **a half cone** (b) **a double cone** (c) **a half-sphere**
(d) **a complete sphere** (e) **a cylinder** (f) **a plane**

#I-2. A table of values is given for a function $f(x, y)$ defined on $R = [1, 3] \times [0, 4]$. Use this table to estimate $\iint f(x, y) dA$ using the **upper right-hand corner** method with $m = n = 2$.

$x \setminus y$	0	1	2	3	4
1.0	2	0	-3	-6	-5
1.5	3	1	-4	-8	-6
2.0	4	3	0	-5	-6
2.5	5	5	3	-1	-4
3.0	7	8	6	3	0

- (a) **-4** (b) **0** (c) **4**
(d) **-2** (e) **2** (f) **8**

#I-3. Find the **y-coordinate** of the center of mass of the thin plate that occupies the region

$$\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$$

and has the density function $\rho(x, y) = x$.

- (a) **1/6** (b) **1/4** (c) **1/3** (d) **1/2** (e) **2/3** (f) **3/4**

#I-4. Rewrite the double integral $\iint_R \left(\frac{x-y}{x+y}\right)^4 dy dx$ using the change of variables $x = \frac{1}{2}(v+u)$, $y = \frac{1}{2}(v-u)$, where R is the triangular region bounded by the line $x+y=1$ and the two coordinate axes.

- (a) $\frac{1}{2} \int_0^1 \int_{-v}^v u^4 v^{-4} du dv$ (b) $2 \int_0^2 \int_{-v}^v u^4 v^{-4} du dv$ (c) $\frac{1}{2} \int_0^2 \int_{-v}^v u^{-4} v^4 du dv$
(d) $\frac{1}{2} \int_0^1 \int_{-v}^v u^4 v^4 du dv$ (e) $2 \int_0^2 \int_{-v}^v u^4 v^4 du dv$ (f) $\frac{1}{2} \int_0^2 \int_{-v}^v u^{-4} v^4 du dv$

PART II: WRITTEN PROBLEMS. To earn full credit for the following problems **you must show your work**.

You can leave answers in terms of fractions and square roots.

#II-1. Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{1-y^2}} dy dx$.

#II-2. Find the volume of the largest rectangular box that can be inscribed inside the ellipsoid $x^2 + 2y^2 + 3z^2 = 6$.

#II-3. (a) Set up the triple integral $\iiint dV$ over the tetrahedron formed by the three coordinate planes and the plane $x + 2y + 3z = 6$ using three different orders of integration (you can pick any three orders you want).

(b) Compute this triple integral using any order of integration.

#II-4. Find the mass of a ball of radius R centered at the origin, where the density at any point P of the ball is proportional to the distance from P to the z -axis.

#II-5. Find the volume of the solid bounded by $x^2 + y^2 = z^2$ and $x^2 + y^2 + z^2 = 8$.
