

MATH 233H FINAL (TAKE HOME)

This exam is worth 100 points, with each problem worth 20 points. Please complete Problem 1 and *any four* of the remaining problems. You must justify your answer to receive credit for a solution; correct answers alone are not necessarily sufficient for credit.

The exam will be submitted on Moodle as individual problems. Please submit exactly five problems (including Problem 1); if you submit more than five (including submitting Problem 1) only the first five (in numerical order) will be graded. If you submit more than four and don't submit Problem 1, then only the first four in numerical order will be graded.

Please make sure your name and student ID are written somewhere in your answers. PDF must be submitted on Moodle; no other formats (such as doc, jpg, tiff, etc) will be accepted. Also please name your PDF files in the form

StudentID_LastName_FirstName_Exam1_ProblemX.pdf

where X is the problem number. For example,

314159_Gunnells_Paul_Exam1_Problem1.pdf

ADDITIONAL INSTRUCTIONS FOR TAKE-HOME EXAM.

The exam answers must be submitted in PDF. Scans of handwriting are ok, but please be sure that they are at a sufficiently high resolution for me to be able to read them. The following **are allowed**:

- You may use class materials (textbook, your own notes, hw assignments, lecture notes from video lectures, video lectures, and other materials on our course pages) during the exam.
- You may use the Desmos Scientific calculator <https://www.desmos.com/scientific> to assist with numerical computations. Algebraic computations must be done by hand. You may also use your own calculator if you prefer it; Desmos is allowed so that everyone is guaranteed to have access to something.

The following **are not allowed**:

- Discussing the exam with anyone in the class or elsewhere. Exception: you may ask me by email for clarification about a problem, just like in the classroom exam. I will try to check email often but unavoidably there will be delays in replies.
- Using any other sources of information (internet, other books, other notes, tables, Wikipedia, etc.) during the exam. In particular you are allowed to look at your own HW, but not any materials away from WebAssign.
- Using a computer (other than Desmos above or for access to video lectures and our course page). In particular programming is not allowed.

When submitting your exam, you are agreeing to the following statement:

I hereby declare that the work submitted represents my individual effort. I have neither given nor received any help and have not consulted any online resources other than those authorized. I attest that I have followed the instructions of the exam.

Academic honesty is very important to me.

- (1) (20 points) Please compute the following. In this problem (and only this problem), there is no partial credit awarded and it is sufficient to just write the answers of the computations.
- (a) (4 points) Let $\vec{v} = \langle 1, 2, -1 \rangle$ and $\vec{w} = \langle 0, 1, -2 \rangle$. Compute $\vec{v} \cdot \vec{w}$ and $\vec{v} \times \vec{w}$.
- (b) (4 points) Let $P = (0, 1, 1)$, $Q = (1, -1, 0)$, $R = (1, 0, 3)$. Find the area of the triangle with vertices at P, Q, R .
- (c) (4 points) Let $g(x, y) = \cos(xy) + x^2y + y^3$. Find an equation for the tangent plane to the graph of g at $(0, 0, 1)$.
- (d) (4 points) Let $\vec{F}(x, y, z) = \langle x^2 + y, z - x, x^2 - z^2 \rangle$. Compute $\text{curl} \vec{F}$ and $\text{div} \vec{F}$.
- (e) (4 points) Compute $\oint_C x \, dy$ where C is the square running from $(0, 0)$ to $(1, 0)$ to $(1, 1)$ to $(0, 1)$ and then back to $(0, 0)$.
- (2) (20 points) Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 12$.
- (3) (20 points) Evaluate $\iiint_E xyz \, dV$, where E is the region between the spheres $\rho = 2$ and $\rho = 4$ and above the cone $\varphi = \pi/3$.
- (4) (20 points) Let $\alpha > 0$ be a constant and let S be the graph of $z = \sqrt{\alpha x^2 + \alpha y^2}$ between $z = 0$ and $z = \sqrt{\alpha}$. Compute the surface area of S .
- (5) (20 points) Let R be the region in the xy plane between the graphs of $xy = 2$, $xy = 6$, $y = 4$ and $y = 10$. Use the change of variables $x = v$, $y = (2u)/(3v)$ to compute

$$\iint_R \frac{15y}{x} \, dA.$$

- (6) (20 points) Let \vec{F} be the vector field $\langle 3x^2e^{2y} + 4ye^{4x}, 1 + 2x^3e^{2y} + e^{4x} \rangle$. Compute $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve along the parabola $y = x^2$ going from the origin to $(1, 1)$.
- (7) (20 points) Let \vec{F} be the vector field $\langle x^3z - 2z, xz, xy \rangle$. Let C the closed triangular curve running along line segments from $(0, 0, 4)$ to $(0, 2, 0)$ to $(2, 0, 0)$ and back to $(0, 0, 4)$. Compute

$$\oint_C \vec{F} \cdot d\vec{r}.$$

- (8) (20 points) Let $\beta > 0$ be a constant and let P be the octahedral surface with vertices $(\beta, 0, 0)$, $(-\beta, 0, 0)$, $(0, \beta, 0)$, $(0, -\beta, 0)$, $(0, 0, \beta)$, $(0, 0, -\beta)$. (see Figure 1.) Assume that P is oriented with the outward pointing normal. Let $\vec{F}(x, y, z) = \langle x - y - z, -x + y - z, -x - y + z \rangle$. Compute the flux of \vec{F} through P :

$$\oiint_P \vec{F} \cdot d\vec{S}.$$

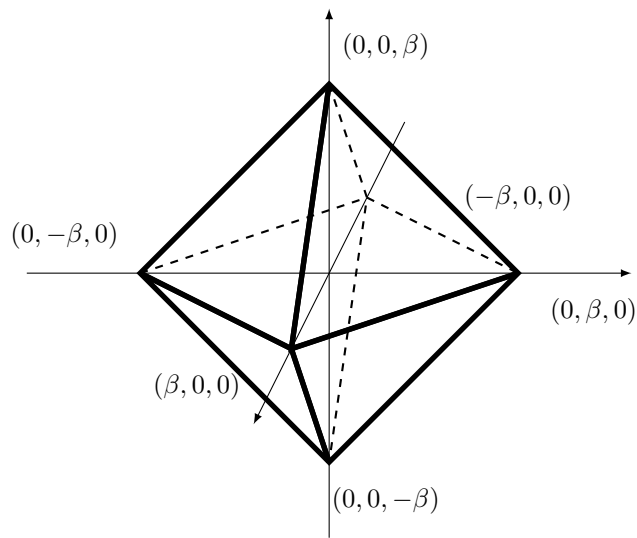


FIGURE 1. The octahedron P