MATH 233H EXAM 2 (TAKE HOME)

This exam is worth 100 points, with each problem worth 20 points. Please complete Problem 1 and *any four* of the remaining problems. You must justify your answer to receive credit for a solution; correct answers alone are not necessarily sufficient for credit.

The exam will be submitted on Moodle as individual problems. Please submit exactly five problems (including Problem 1); if you submit more than five (including submitting Problem 1) only the first five (in numerical order) will be graded. If you submit more than four and don't submit Problem 1, then only the first four in numerical order will be graded.

Please make sure your name and student ID are written somewhere in your answers. PDF must be submitted on Moodle; no other formats (such as doc, jpg, tiff, etc) will be accepted. Also please name your PDF files in the form

`StudentID_LastName_FirstName_Exam1_ProblemX.pdf`

where X is the problem number. For example,

`314159_Gunnells_Paul_Exam1_Problem1.pdf`

ADDITIONAL INSTRUCTIONS FOR TAKE-HOME EXAM.

The exam answers must be submitted in PDF. Scans of handwriting are ok, but please be sure that they are at a sufficiently high resolution for me to be able to read them. The following **are allowed**:

- You may use class materials (textbook, your own notes, hw assignments, lecture notes from video lectures, video lectures, and other materials on our course pages) during the exam.
- You may use the Desmos Scientific calculator `https://www.desmos.com/scientific` to assist with numerical computations. Algebraic computations must be done by hand. You may also use your own calculator if you prefer it; Desmos is allowed so that everyone is guaranteed to have access to something.

The following **are not allowed**:

- Discussing the exam with anyone in the class or elsewhere. Exception: you may ask me by email for clarification about a problem, just like in the classroom exam. I will try to check email often but unavoidably there will be delays in replies.
- Using any other sources of information (internet, other books, other notes, tables, Wikipedia, etc.) during the exam. In particular you are allowed to look at your own HW, but not any materials away from WebAssign.
- Using a computer (other than Desmos above or for access to video lectures and our course page). In particular programming is not allowed.

When submitting your exam, you are agreeing to the following statement:

I hereby declare that the work submitted represents my individual effort. I have neither given nor received any help and have not consulted any online resources other than those authorized. I attest that I have followed the instructions of the exam.

Academic honesty is very important to me.

Date: Assigned: Tuesday 20 Oct 2020; Due: on Moodle by 6pm, Thursday 22 Oct 2020.

(1) (20 points) Please compute the following. In this problem (and only this problem), there is no partial credit awarded and it is sufficient to just write the answers of the computations.

(a) (4 points) Let $f(x,y) = xy^2 \sin(x^3 + y^2)$. Compute f_x and f_y.

(b) (4 points) Let $g(x,y) = x^2y + e^{x^2-y^2}$. Compute g_{xx}, g_{xy}, and g_{yy}.

(c) (4 points) Let $h(x,y,z) = x^2z + x^3y + xyz^2$. Compute ∇h.

(d) (4 points) Let $t(x,y) = x^3y^2 - xy^3$. Compute the Hessian D of t as a function of x and y.

(e) (4 points) Let $u(x,y,z) = x^2y + xz^2 + yz$. Compute the directional derivative of u at $(1,2,3)$ in the direction of the vector $\langle 2,6,9\rangle$.

(2) (20 points) Let H be the hyperboloid of one sheet given by $x^2 + y^2 - z^2 = 1$. Find all points on H where the tangent plane is parallel to the plane $6x + 7y + 9z = 0$, or show that there are no such points.

(3) (20 points) Let D be the closed triangle in the xy-plane with vertices $(-2,0)$, $(2,0)$, and $(0,2)$. Find the absolute maximum and the absolute minimum of $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$ on D.

(4) (20 points) Find and classify all the critical points of $f(x,y) = x^3 - 3x^2 + 3xy^2 + 3y^2$.

(5) (20 points) Suppose that $0 < a < b < c$ are three fixed numbers satisfying $ab + ac + bc = 1$. Find the maximum and minumum value of $2x + 2y + 2z$ on the surface $ax^2 + by^2 + cz^2 = 1$.

(6) (20 points) Compute

$$\iint_R e^{y^3}\, dA$$

where R is the region in the xy-plane bounded by the y-axis, $y = 1$, and $y = \sqrt{x}$.

(7) (20 points) Let R_1 be the region $x^2 + y^2 \leq 1$ and let R_2 be the region bounded by the graph of $r = 1 + \cos\theta$. Compute the area of the intersection of R_1 and R_2.