MATH 233H EXAM I (TAKE HOME)

This exam is worth 100 points, with each problem worth 20 points. Please complete Problem 1 and *any four* of the remaining problems. You must justify your answer to receive credit for a solution; correct answers alone are not necessarily sufficient for credit.

The exam will be submitted on Moodle as individual problems. Please submit exactly five problems (including Problem 1); if you submit more than five (including submitting Problem 1) only the first five (in numerical order) will be graded. If you submit more than four and don't submit Problem 1, then only the first four in numerical order will be graded.

Please make sure your name and student ID are written somewhere in your answers. PDF must be submitted on Moodle; no other formats (such as doc, jpg, tiff, etc) will be accepted. Also please name your PDF files in the form

`StudentID_LastName_FirstName_Exam1_ProblemX.pdf`

where X is the problem number. For example,

`314159_Gunnells_Paul_Exam1_Problem1.pdf`

ADDITIONAL INSTRUCTIONS FOR TAKE-HOME EXAM.

The exam answers must be submitted in PDF. Scans of handwriting are ok, but please be sure that they are at a sufficiently high resolution for me to be able to read them. The following **are allowed**:

- You may use class materials (textbook, your own notes, hw assignments, lecture notes from video lectures, video lectures, and other materials on our course pages) during the exam.
- You may use the Desmos Scientific calculator `https://www.desmos.com/scientific` to assist with numerical computations. Algebraic computations must be done by hand. You may also use your own calculator if you prefer it; Desmos is allowed so that everyone is guaranteed to have access to something.

The following **are not allowed**:

- Discussing the exam with anyone in the class or elsewhere. Exception: you may ask me by email for clarification about a problem, just like in the classroom exam. I will try to check email often but unavoidably there will be delays in replies.
- Using any other sources of information (internet, other books, other notes, tables, Wikipedia, etc.) during the exam. In particular you are allowed to look at your own HW, but not any materials away from WebAssign.
- Using a computer (other than Desmos above or for access to video lectures and our course page). In particular programming is not allowed.

When submitting your exam, you are agreeing to the following statement:

I hereby declare that the work submitted represents my individual effort. I have neither given nor received any help and have not consulted any online resources other than those authorized. I attest that I have followed the instructions of the exam.

Academic honesty is very important to me.

Date: Assigned: Tuesday 22 September 2020; Due: on Moodle by 6pm, Thursday 24 September 2020.

(1) (20 points) Let $\vec{a} = \langle 1, 1, 1\rangle$, $\vec{b} = \langle 1, 2, 3\rangle$, $\vec{c} = \langle 1, -2, 1\rangle$. Please compute the following. In this problem (and only this problem), there is no partial credit awarded and it is sufficient to just write the answers of the computations.
   (a) (4 points) $\vec{a} \cdot \vec{b}$.
   (b) (4 points) $\vec{b} \times \vec{c}$.
   (c) (4 points) The angle between the vectors $\vec{a}$ and $\vec{c}$.
   (d) (4 points) The area of the triangle T with vertices at the tips of $\vec{a}$, $\vec{b}$, $\vec{c}$.
   (e) (4 points) An equation for the plane containing the triangle T from above.

(2) (20 points) Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS, where the points P, Q, R, S have coordinates

$$P(3, 2, 1), \quad Q(3, 3, 0), \quad R(4, 2, 2), \quad S(4, 4, 3).$$

(3) (20 points) Let $\vec{r}_1 = \langle 4 + 2t, t^2, -t\rangle$, $\vec{r}_2(s) = \langle 1 + s, s, s^2\rangle$ be two space curves, where t, s range over all real numbers.
   (a) (6 points) Determine all the points of intersection of these space curves.
   (b) (7 points) At any point of intersection, find parametric equations for the tangent lines to the space curves. (In other words, for any intersection point P, there is a tangent line to $\vec{r}_1$ at P and a tangent line to $\vec{r}_2$ at P, and you should compute both equations.) .
   (c) (7 points) At any point of intersection, determine the angle between the tangent lines at that point.

(4) (20 points) Two lines are given by the vector-valued functions

$$\vec{r}_1(t) = \langle 2t + 1, t, -3t + 2\rangle, \quad \vec{r}_2(s) = \langle -4s, -2s + 1, 6s\rangle,$$

where t, s vary over all real numbers.
   (a) (8 points) Determine if the lines intersect, are parallel, or are skew.
   (b) (12 points) If the lines intersect, find the point of intersection. Otherwise, find the distance between the two lines.

(5) (20 points) Let S be the graph of $x^2 + z^2 = 4$ and T the graph of $y = x^2 - 2z^2$. Determine a vector-valued function for the curve given by the intersection of S and T.

(6) (20 points) A particle moves along the graph of $\vec{r}(t) = \langle \cos(t^3), \sin(t^3), t^3\rangle$ for all real numbers t.
   (a) (4 points) Compute the velocity of the particle.
   (b) (2 points) Compute the speed of the particle.
   (c) (6 points) Compute the acceleration of the particle.
   (d) (8 points) Compute the distance traveled by the particle from $t = -2\pi$ to $t = 2\pi$.

(7) (20 points) Let $\vec{a} = \langle a_1, a_2, a_3\rangle$, $\vec{b} = \langle b_1, b_2, b_3\rangle$, $\vec{c} = \langle c_1, c_2, c_3\rangle$. Show that $\vec{a} \times (\vec{b} \times \vec{c}) = \alpha\vec{b} - \beta\vec{c}$, where $\alpha = \vec{a} \cdot \vec{c}$ and $\beta = \vec{a} \cdot \vec{b}$. (*Hint:* Just compute everything.)