Practice problems from old exams for math 233

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October 26, 2012

Disclaimer: Your instructor covers far more materials that we can possibly fit into a four/five questions exams. These practice tests are meant to give you an idea of the kind and varieties of questions that were asked within the time limit of that particular tests. In addition, the scope, length and format of these old exams might change from year to year. Users beware! These are NOT templates upon which future exams are based, so don't expect your exam to contain problems exactly like the ones presented here. Check the course web page for an update on the material to be covered on each exam or ask your instructor.

1 Practice problems for Exam 1.

Fall 2008 Exam

1. (a) Find parametric equations for the line **L** which contains $A(1, 2, 3)$ and $B(4, 6, 5)$.

   (b) Find parametric equations for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

2. (a) Find an equation of the plane which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

   (b) Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

   (c) Find the point Q in the plane $2x + y - 2z = 19$ which is closest to the point $(1, 6, -1)$. (Hint: You can use part b) of this problem to help find Q or first find the equation of the line **L** passing through Q and the point $(1, 6, -1)$ and then solve for Q.)

3. (a) Find the volume **V** of the parallelepiped such that the following four points $A = (3, 4, 0)$, $B = (3, 1, -2)$, $C = (4, 5, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

   (b) Find the center and radius of the sphere $x^2 - 4x + y^2 + 4y + z^2 = 8$.

4. (a) The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
   Find an equation of the tangent line to the curve at the point $(4, -4, 2)$.

   (b) Find the length **L** of the arc traveled from time $t = 1$ to time $t = 4$.

(c) Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, 2\cos 2t, 3e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

5. (a) Consider the points $A(2,1,0)$, $B(3,0,2)$ and $C(0,2,1)$. Find the area of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.

   (b) Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(2,1,1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Spring 2008 Exam

6. (a) Find an equation of the plane through the points $A = (1,2,3)$, $B = (0,1,3)$, and $C = (2,1,4)$.

   (b) Find the area of the triangle with vertices at points A, B, and C given above. *Hint: the area of this triangle is related to the area of a certain parallelogram*

7. (a) Find the parametric equations of the line passing through the point $(2,4,1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

   (b) Find the intersection point of the line in part (a) and the plane $3x - y + 5z = 77$.

8. (a) A *plane* curve is given by the graph of the vector function

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \le t \le 2\pi.$$

   Find a single equation for the curve in terms of x and y, by eliminating t.

   (b) Consider the *space* curve given by the graph of the vector function

$$\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle, \quad 0 \le t \le 2\pi.$$

   Sketch the curve and indicate the direction of increasing t in your graph.

   (c) Determine parametric equations for the line T tangent to the graph of the *space* curve for $\mathbf{r}(t)$ at $t = \pi/3$, and sketch T in the graph obtained in part **(b)**.

9. Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle$.

   (a) Determine $\mathbf{r}(t)$ for all t.

(b) Show that $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}'(t)$ for all t.

(c) Find the arclength of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \le t \le 1$.

10. If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \ge 0$ is measured in seconds and distance is measured in feet),

    (a) Find the speed and the velocity of the object at time t.

    (b) If a second object travels along a path given defined by the graph of the vector function $\mathbf{w}(s) = \langle 2, 5, 1\rangle + s\langle 2, -1, -5\rangle$, show that the paths of the two objects intersect at a common point P.

    (c) If $s = t$ in part **(b)**, (i.e. the position of the second object is $\mathbf{w}(t)$ when the first object is at position $\mathbf{r}(t)$), do the two objects ever collide?

Spring 2007 Exam

11. (a) Find parametric equations for the line which contains $A(7, 6, 4)$ and $B(4, 6, 5)$.

    (b) Find the parametric equations for the line of intersection of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

12. (a) Find an equation of the plane which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

    (b) Find the distance from the point $(1, 0, -1)$ to the plane $2x + y - 2z = 1$.

    (c) Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

13. (a) Consider the two space curves

    $$\mathbf{r}_1(t) = \langle \cos(t-1), t^2 - 1, 2t^4\rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, 2s^2\rangle,$$

    where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1, 0, 2)$.

    (b) Find the center and radius of the sphere $x^2 + y^2 + 2y + z^2 + 4z = 20$.

14. The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \ge 0$.

    (a) At the time $t = 4$, this particle is at the point $(0, 5, 4)$. Find an equation of the tangent line to the curve at the time $t = 4$.

(b) Find the length of the arc traveled from time $t = 2$ to time $t = 4$.

(c) Find a vector function which represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + 2y + z = 4$.

15. (a) Consider the points $A(2, 1, 0)$, $B(1, 0, 2)$ and $C(0, 2, 1)$. Find the area of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

(b) Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Fall 2007 Exam

16. Find the equation of the plane containing the lines

$$x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and}$$
$$x = 4 - t, \quad y = 3 + 2t, \quad z = 1$$

17. Find the distance between the point $P(3, -2, 7)$ and the plane given by

$$4x - 6y - z = 5.$$

18. Determine whether the lines L_1 and L_2 given below are parallel, skew or intersecting. If they intersect, find the point of intersection.

$$L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

19. (a) Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

Find the acceleration of the particle. Write down a formula for the speed of the particle (you do not need to simplify the expression algebraically).

(b) If initially the particle has the position $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$, what is the position at time t?

20. Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are PQ and PR. This problem continues on the next page.

(a) Find the area of the parallelogram.

(b) Find the cosine of the angle between the vector $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(c) Find the coordinates of the fourth vertex.

21. Let C be the parametric curve

$$x = 2 - t^2, y = 2t - 1, z = \ln t.$$

This problem continues on the next page.

(a) Determine the point(s) of intersection of C with the xz-plane.

(b) Determine the parametric equation of the tangent line to C at $(1, 1, 0)$.

(c) Set up, but not solve, a formula that will determine the length of C for $1 \leq t \leq 2$.

Fall 2006 Exam

22. (a) Find parametric equations for the line which contains $A(2, 0, 1)$ and $B(-1, 1, -1)$.

(b) Determine whether the lines $l_1 : x = 1 + 2t,\ y = 3t, z = 2 - t$ and $l_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are parallel, skew or intersecting.

23. (a) Find an equation of the plane which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

(b) Find the distance from the point $(1, 2, -1)$ to the plane $2x+y-2z = 1$.

24. (a) Let two space curves

$$\mathbf{r}_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle,$$

be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1, 0, 1)$.

(b) Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

25. (a) Let $f(x, y) = e^{x^2 - y} + x\sqrt{4 - y^2}$. Find partial derivatives f_x, f_y and f_{xy}.

(b) Find an equation for the tangent plane of the graph of

$$f(x,y) = \sin(2x+y)+1$$

at the point $(0,0,1)$.

26. (a) Let $g(x,y) = ye^x$. Estimate $g(0.1, 1.9)$ using the linear approximation of $g(x,y)$ at $(x,y) = (0,2)$.
    (b) Find the center and radius of the sphere $x^2+y^2+z^2+6z = 16$.
    (c) Let $f(x,y) = \sqrt{16-x^2-y^2}$. Draw a contour map of level curves $f(x,y) = k$ with $k = 1,2,3$. Label the level curves by the corresponding values of k.

These problems are from older exams

27. Consider the line L through points $A = (2,1,-1)$ and $B = (5,3,-2)$. Find the intersection of the line L and the plane given by $2x - 3y + 4z = 13$.

28. Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2\rangle,\ \mathbf{r}_2(s) = \langle 3-s, s-2, s^2\rangle$$

where t and s are two independent real parameters.

(a) Show that the two space curves intersect by finding the point of intersection and the parameter values where this occurs.

(b) Find parametric equation for the tangent line to the space curve $\mathbf{r}(t)$ at the intersection point.

29. Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A. If $A = (2,5,1)$, $B = (3,1,4)$, $D = (5,2,-3)$, find the point C.

30. Consider the points $A = (2,1,0)$, $B = (1,0,2)$ and $C = (0,2,1)$.

(a) Find the orthogonal projection $proj_{\overrightarrow{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

(b) Find the area of triangle ABC.

(c) Find the distance d from the point C to the line L that contains points A and B.

31. Find parametric equations for the line of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

32. Let L_1 denote the line through the points $(1,0,1)$ and $(-1,4,1)$ and let L_2 denote the line through the points $(2,3,-1)$ and $(4,4,-3)$. Do the lines L_1 and L_2 intersect? If not, are they skew or parallel?

33. (a) Find the volume of the parallelepiped such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

    (b) Find an equation of the plane through A, B, D.

    (c) Find the angle between the plane through A, B, C and the xy plane.

34. The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.

    (a) At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.

    (b) Find an equation of the tangent line to the curve at the time $t = 4$.

    (c) Does the particle ever pass through the point $P = (80, 41, 13)$?

    (d) Find the length of the arc traveled from time $t = 1$ to time $t = 2$.

35. Consider the surface $x^2 + 3y^2 - 2z^2 = 1$.

    (a) What are the traces in $x = k, y = k, z = k$? Sketch a few.

    (b) Sketch the surface in the space.

36. Find an equation for the tangent plane to the graph of $f(x, y) = y \ln x$ at $(1, 4, 0)$.

37. Find the distance between the given parallel planes

$$z = 2x + y - 1, \quad -4x - 2y + 2z = 3.$$

38. Identify the surface given by the equation $4x^2 + 4y^2 - 8y - z^2 = 0$. Draw the traces and sketch the curve.

39. A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.

    a) Write an equation for the acceleration vector.
    b) Write a vector for initial velocity.
    c) Write a vector for initial position.
    d) At what time does the projectile hit the ground?
    e) How far did it travel, horizontally, before it hit the ground?

40. Explain why the limit of $f(x, y) = (3x^2y^2)/(2x^4 + y^4)$ does not exist as (x, y) approaches $(0, 0)$.

41. Find an equation of the plane that passes through the point $P(1, 1, 0)$ and contains the line given by parametric equations $x = 2+3t,\ y = 1-t$, $z = 2 + 2t$.

42. Find all of the first order and second order partial derivatives of the function.

    (a) $f(x, y) = x^3 - xy^2 + y$

(b) $f(x, y) = \ln(x + \sqrt{x^2 + y^2})$

43. Find the linear approximation of the function $f(x, y) = xye^x$ at $(x, y) = (1, 1)$, and use it to estimate $f(1.1, 0.9)$.

44. Find a vector function which represents the curve of intersection of the paraboloid $z = 2x^2 + y^2$ and the parabolic cylinder $y = x^2$.

Spring 2009 Exam

45. Given $\vec{a} = \langle 3, 6, -2 \rangle$, $\qquad \vec{b} = \langle 1, 2, 3 \rangle$.

    a) Write down the vector projection of $\vec{\mathbf{b}}$ to $\vec{\mathbf{a}}$. (Hint: Use projections.)

    b) Write $\vec{\mathbf{b}}$ as a sum of a vector parallel to $\vec{\mathbf{a}}$ and a vector orthogonal to $\vec{\mathbf{a}}$. (Hint: Use projections.)

    c) Let θ be the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. Find $\cos\theta$.

46. Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.

    a) Let L be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for L.

    (b) A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

47. Consider the points $P(1, 3, 5)$, $Q(-2, 1, 2)$, $R(1, 1, 1)$ in $\mathbb{R}^3$.

    a) Find an equation for the plane containing P, Q and R.

    b) Find the area of the triangle with vertex P, Q, R.

48. Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

49. Consider the parameterized curve

$$\mathbf{r}(t) = \langle t, t^2, t^3 \rangle, \ t \in \mathbb{R}.$$

    a) Set up an integral for the length of the arc between $t = 0$ and $t = 1$. Do **not** attempt to evaluate the integral.

    b) Write down the parametric equations of tangent line to $\mathbf{r}(t)$ at $(2, 4, 8)$.

50. a) Consider the sphere $\mathbf{S}$ in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 + z^2 - 4x - 6z - 3 = 0.$$

    Find its center $\mathbf{C}$ and its radius $\mathbf{R}$. b) What does the equation $x^2+z^2 = 4$ describe in $\mathbb{R}^3$? Make a sketch.

51. Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration on the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball. Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and y represents the height.

(b) Find the speed of the ball at its highest point.

(c) At what time T does the ball reach its highest point.

Fall 2009 Exam

52. Let $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 6\mathbf{k} + 9\mathbf{j}$.

(a) Find the vector representing the projection of $\mathbf{v}$ onto $\mathbf{w}$.

(b) Find $\cos\theta$, where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$.

53. Consider the points $P = (0, 3, -3)$, $Q = (-1, 3, 2)$, $R = (-1, 2, -3)$.

(a) Find an equation for the plane containing P, Q, R.

(b) Find the area of the triangle with vertices P, Q, R.

54. Let P_1 be the plane $x + y - z = 0$ and P_2 be the plane $x - 2y + z = 1$.

(a) Find parametric equations for the line of intersection of P_1 and P_2.

(b) Find the distance from the origin to the plane P_2.

55. Let $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + t^6\mathbf{k}$.

(a) Find an equation for the tangent line to the graph at the point given by $t = 1$.

(b) Find the unit tangent vector $\mathbf{T}$ to the graph at the point given by $t = 1$.

(c) Write a definite integral that computes the length of the graph of $\mathbf{r}(t)$ from $t = 1$ to $t = 2$, but do **not** attempt to evaluate it.

56. Consider a particle moving with acceleration $\mathbf{a}(t) = \langle t, e^t, -\sin(t)\rangle$.

(a) Find the velocity vector $\mathbf{v}(t)$ of the particle, assuming that $\mathbf{v}(0) = \mathbf{0}$.

(b) Find the position vector $\mathbf{r}(t)$ of the particle, assuming that $\mathbf{r}(0) = \mathbf{0}$.

Spring 2010 Exam

57. Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A where $A = (1, 2, -1)$, $B = (3, 5, 1)$ and $D = (2, -1, 2)$.

(a) Find the area of the parallelogram.

(b) Find the coordinates of the point C.

58. Consider the points $A = (0, 3, -3)$, $B = (-1, 3, 2)$, $C = (-1, 2, -3)$.

(a) Find the orthogonal projection $proj_{\overrightarrow{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

(b) Find the distance d from the point C to the line L that contains points A and B.

59. Let P_1 be the plane $x + 3y + z = 0$ and P_2 be the plane $2x + y - z = 1$.

    (a) Find the cosine of the angle between the planes.

    (b) Find the parametric equations of the line of intersection between the 2 planes P_1 and P_2.

    (c) Find the distance from the plane P_2 to the origin.

60. Let $\mathbf{r}(t) = \cos(2t)\mathbf{i} + \sin(2t)\mathbf{j} + t\mathbf{k}$.

    (a) What is the length of the curve starting at $t = 0$ and ending at $t = 5$.

    (b) Find a vector equation for the tangent line to the graph at the point given by $t = 0$.

61. Show that the limit $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist.

62. Consider the sphere $\mathbf{S}$ in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 + z^2 - 2x - 8y - 2 = 0$$

    (a) Find the coordinates of its center and its radius.

    (b) What does the equation $x^2+y^2 = 64$ describe in $\mathbb{R}^3$. Make a sketch.

Spring 2012 Exam

63. Given $A = (-1, 7, 5)$, $B = (3, 0, 2)$ and $C = (1, 2, 3)$.

    (a) Let L be the line which passes through the points A and B. Find the parametric equations for L.

    (b) A, B and C are three of the four vertices of a parallelogram, while AB and BC are two of the four edges. Find the fourth vertex.

64. Consider the points $P(1, 1, 1)$, $Q(-2, 1, 2)$, $R(1, 3, 5)$ in $\mathbb{R}^3$.

    (a) Find an equation for the plane containing P, Q and R.

    (b) Find the area of the triangle with vertices P, Q, R.

65. Let P_1 be the plane $x-2y+2z = 10$ and P_2 be the plane $2x+y+2z = 0$. Find the cosine of the angle between the planes.

66. a) Find the distance from the point $Q = (1, 6, -1)$ to the plane $2x + y - 2z = 19$. (Hint: To do this you can use the vector projection of some vector $\vec{PQ}$, where P is some point on the plane.)

    b) Write the parametric equations of the line L containing the point $T(1, 2, 3)$ and perpendicular to the plane $2x + y - 2z = 19$.

    c) Find the point of intersection of the line L in part b) with the plane $2x + y - 2z = 19$.

67. Find the equation of the sphere with center at the point $(1, 2, 3)$ and which contains the point $(3, 1, 5)$.

68. Make a sketch of the surface in $\mathbb{R}^3$ described by equation $y^2 + z^2 = 36$. In your sketch of this surface, include the labeled coordinate axes and the trace curves on the surface for the planes $x = 0$ and $x = 4$.

69. Find the equation of the plane which contains the points $A(1, 2, 3)$ and $B(1, 0, 4)$ and which is also perpendicular to the plane $4x - 2y + z = 8$.

70. Suppose $\vec{a} = \langle 2, 1, 2 \rangle$ and $\vec{b} = \langle 8, 2, 0 \rangle$.

    (a) Find the vector projection, call it $\vec{c}$, of $\vec{b}$ in the direction $\vec{a}$.

    (b) Calculate the vector $\vec{b} - \vec{c}$ and then show that it is orthogonal to $\vec{a}$.