MATH 233 PRACTICE MT#1, VERSION #1

DISCLAIMER: This practice exam is intended to give you an *idea* about what a two-hour midterm is like. It is not possible for any one exam to cover every topic, and the *content, coverage and format of your actual exam could be different from this practice exam.*

You can leave answers in terms of fractions and square roots.
To earn full credit you must show your work.

#1. (a) Determine parametric equations for the line L_2 tangent to the space curve given by $\vec{r}(t) = \langle t, 2t^2, 2t\rangle$ at the point $(1, 2, 2)$.
(b) Find the area of the triangle with vertices $A = (1, 1, 3), B = (2, 5, 2)$ and $C = (1, 2, 6)$.

#2. Find the cosine of the angle θ between the xz-plane and the tangent plane of the level surface of $f(x, y, z) = x/y + y/z$ at $(1, 2, 3)$.

#3. Make a sketch of the surface in R^3 described by equation $x^2 - y + z^2 = 0$. In your sketch of this surface, include the labeled coordinate axes and draw and label the trace curves (also known as cross sections) on the surface for the planes $z = 0$ and $z = 3$.

#4. A particle moves along the curve $\vec{r}(t) = \langle t^3/3, t^2, 2t\rangle$. Find the length of the path traveled by the particle between $t = 1$ and $t = 3$.

#5. Consider the function $f(x, y) = \dfrac{x^3 y}{x^6 + y^2}$.
(a) Find the limit of $f(x, y)$ as (x, y) approaches the origin along a straight line of slope m, where m is a real number.
(b) Find the limit of $f(x, y)$ as (x, y) approaches the origin along the curve $y = x^3$.
(c) Does $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ exist? Explain why or why not.

#6. Find the linear approximation to $f(x, y) = \sqrt{xy + 1}$ at the point $(4, 6)$, and use this to estimate $f(3.9, 5.9)$.

#7. Find the direction(s) in which the directional derivative of $f(x, y) = x^2 + \sin(xy)$ at the point $(1, 0)$ has the value 1.

#8. Find all local maxima, minima, and saddle points of the function $f(x, y) = x^3 - 3x + x^2y^2$. Be sure to specify the type of each point you find.
