Homework Set HW6 due 05/04/2013 at 08:00pm EDT

This is a demonstration set designed to show you some types of questions that can be asked using WeBWorK.

You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer.

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1. **(1 pt) Library/FortLewis/DiffEq/4-Laplace-transforms/03-Partial-fractions/KJ-5-3-12.pg**

Consider the function $F(s) = \dfrac{4s-5}{s^2-3s+2}$.

(1) Find the partial fraction decomposition of $F(s)$:

$$\frac{4s-5}{s^2-3s+2} = \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_$$

(2) Find the inverse Laplace transform of $F(s)$.

$f(t) = \mathcal{L}^{-1}\{F(s)\} =$ ____________________

Correct Answers:

- `3/(s-2)`
- `1/(s-1)`
- `3*e^(2*t) + 1*e^(1*t)`

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2. **(1 pt) Library/FortLewis/DiffEq/4-Laplace-transforms/03-Partial-fractions/KJ-5-3-24.pg**

Consider the initial value problem

$$y'' + 16y = \cos(4t), \qquad y(0) = 6, \quad y'(0) = 7.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

________________ = ________________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ____________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ____________________

Correct Answers:

- `s^2*Y(s)-6*s-7+16*Y(s)`
- `s/(s^2+16)`
- `s/[(s^2+16)^2]+(6*s+7)/(s^2+16)`
- `t/8*sin(4*t)+6*cos(4*t)+1.75*sin(4*t)`

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3. **(2 pts) Library/FortLewis/DiffEq/4-Laplace-transforms/03-Partial-fractions/KJ-5-3-29.pg**

Consider the initial value problem

$$y'' + 4y = g(t), \qquad y(0) = 0, \quad y'(0) = 0,$$

where $g(t) = \begin{cases} t & \text{if } 0 \le t < 9 \\ 0 & \text{if } 9 \le t < \infty. \end{cases}$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

________________ = ________________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ____________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ____________________

Correct Answers:

- `s^2*Y(s)+4*Y(s)`
- `1/(s^2)-e^(-9*s)/(s^2)-9*e^(-9*s)/s`
- `1/[s^2*(s^2+4)]-e^(-9*s)/[s^2*(s^2+4)] -9*e^(-9*s)/[s*(s^2+4)]`
- `0.25*[t-0.5*sin(2*t)-(t-9)*h(t-9) +0.5*sin(2*(t-9))*h(t-9)] +(-2.25*[h(t-9)-h(t-9)*cos(2*(t-9))])`

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4. **(2 pts) Library/FortLewis/DiffEq/4-Laplace-transforms/03-Partial-fractions/KJ-5-3-23.pg**

Consider the initial value problem

$$y'' + 9y = 36t, \qquad y(0) = 6, \quad y'(0) = 2.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

________________ = ________________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ____________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ____________________

Correct Answers:

- s^2*Y(s)-6*s-2+9*Y(s)
- 36/(s^2)
- 36/[s^2*(s^2+9)]+(6*s+2)/(s^2+9)
- 4*t-1.33333*sin(3*t)+6*cos(3*t)+0.666667*sin(3*t)

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5. **(2 pts) Library/FortLewis/DiffEq/4-Laplace-transforms/07-Delta-function/KJ-5-7-15.pg**

Consider the following initial value problem, in which an input of large amplitude and short duration has been idealized as a delta function.

$$y'' + 16\pi^2 y = 4\pi\delta(t-5), \qquad y(0) = 0, \quad y'(0) = 0.$$

(1) Find the Laplace transform of the solution.

$Y(s) = \mathcal{L}\{y(t)\} =$ ____________

(2) Obtain the solution $y(t)$.

$y(t) =$ ____________

(3) Express the solution as a piecewise-defined function and think about what happens to the graph of the solution at $t = 5$.

$$y(t) = \begin{cases} \_\_\_\_\_\_ & \text{if } 0 \le t < 5, \\ \_\_\_\_\_\_ & \text{if } 5 \le t \le \infty. \end{cases}$$

Correct Answers:

- 4*pi*e^(-5*s)/(s^2+16*pi^2)
- h(t-5)*sin(4*pi*(t-5))
- 0
- sin(4*pi*(t-5))

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6. **(2 pts) Library/FortLewis/DiffEq/4-Laplace-transforms/02-Shifts-and-IVPs/KJ-5-2-42.pg**

Consider the initial value problem

$$y' + 6y = \begin{cases} 0 & \text{if } 0 \le t < 2 \\ 11 & \text{if } 2 \le t < 7 \\ 0 & \text{if } 7 \le t < \infty, \end{cases} \qquad y(0) = 9.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

____________ = ____________

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ____________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ____________

Correct Answers:

- s*Y(s)-9+6*Y(s)
- 11*[e^(-2*s)/s-e^(-7*s)/s]
- (11*[e^(-2*s)/s-e^(-7*s)/s]+9)/(s+6)
- 1.83333*[h(t-2)-h(t-2)*e^[-6*(t-2)]-h(t-7)+h(t-7)*e^[-6*(t-7)]]+9*e^(-6*t)

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7. **(1 pt) Library/FortLewis/DiffEq/4-Laplace-transforms/02-Shifts-and-IVPs/KJ-5-2-19.pg**

Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{7s-11}{s^2-4s+20}$.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{7s-11}{s^2-4s+20}\right\} =$ ____________

Correct Answers:

- 7*e^(2*t)*cos(4*t)+0.75*e^(2*t)*sin(4*t)

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8. **(1 pt) Library/FortLewis/DiffEq/4-Laplace-transforms/02-Shifts-and-IVPs/KJ-5-2-10.pg**

Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = e^{8t}\cos(4t)$, defined on the interval $t \ge 0$.

$F(s) = \mathcal{L}\{e^{8t}\cos(4t)\} =$ ____________

Correct Answers:

- (s-8)/[(s-8)^2+16]

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9. **(3 pts) Library/FortLewis/DiffEq/4-Laplace-transforms/01-Laplace-transforms/KJ-5-1-10.pg**

(1) Set up an integral for finding the Laplace transform of the following function:

$$f(t) = \begin{cases} 0, & 0 \le t < 6 \\ t-9, & 6 \le t. \end{cases}$$

$F(s) = \mathcal{L}\{f(t)\} = \int_A^B$ ____________

where $A =$ ___ and $B =$ ___.

(2) Find the antiderivative (with constant term 0) corresponding to the previous part.

____

(3) Evaluate appropriate limits to compute the Laplace transform of $f(t)$:

$F(s) = \mathcal{L}\{f(t)\} =$ ____________

(4) Where does the Laplace transform you found exist? In other words, what is the domain of $F(s)$?

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Correct Answers:

- (t-9) * e^(-s*t) * dt
- 6

- `INFINITY`
- `-t/s*e^(-s*t) - 1/(s^2)*e^(-s*t) + 9/s*e^(-s*t)`
- `((6 - 9)/s + 1/(s^2)) * e^(-s*6)`
- `s > 0`

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10. **(1 pt) Library/FortLewis/DiffEq/4-Laplace-transforms/01-Laplace-transforms/KJ-5-1-38.pg**

Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \frac{16s}{s^2-25}$.

$$f(t) = \mathcal{L}^{-1}\left\{\frac{16s}{s^2-25}\right\} = \mathcal{L}^{-1}\left\{\frac{8}{s+5} + \frac{8}{s-5}\right\} = \_\_\_\_\_\_\_\_\_\_$$

Correct Answers:

- `8*e^(-5*t)+8*e^(5*t)`

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11. **(3 pts) Library/FortLewis/DiffEq/4-Laplace-transforms/01-Laplace-transforms/KJ-5-1-21.pg**

From a table of integrals, we know that for $a, b \neq 0$,

$$\int e^{at}\cos(bt)\,dt = e^{at}\cdot\frac{a\cos(bt)+b\sin(bt)}{a^2+b^2} + C.$$

(1) Use this antiderivative to compute the following improper integral:

$$\int_0^\infty e^{4t}\cos(3t)\,e^{-st}\,dt \;=\; \lim_{T\to\infty} \_\_\_\_\_\_\_\_\_\_ \text{ if } s \neq 4$$

or

$$\int_0^\infty e^{4t}\cos(3t)\,e^{-st}\,dt \;=\; \lim_{T\to\infty} \_\_\_\_\_\_\_\_\_\_ \text{ if } s = 4.$$

(2) For which values of s do the limits above exist? In other words, what is the domain of the Laplace transform of $e^{4t}\cos(3t)$?

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(3) Evaluate the existing limit to compute the Laplace transform of $e^{4t}\cos(3t)$ on the domain you determined in the previous part:

$$F(s) = \mathcal{L}\left\{e^{4t}\cos(3t)\right\} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Correct Answers:

- `e^((4-s)*T)*((4-s)*cos(3*T) + 3*sin(3*T))/((4-s)^2+3^2) + (s-4)/((s-4)^2+3^2)`
- `1/3*sin(3*T)`
- `s > 4`
- `(s-4)/[(s-4)^2+9]`

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12. **(1 pt) Library/274/Laplace4/prob51.pg**

Use the Laplace transform to solve the following initial value problem:

$$y'' - 5y' - 14y = \delta(t-8) \qquad y(0) = 0,\; y'(0) = 0$$

Use step(t-c) for $u_c(t)$. $y(t) =$ ____________________.

Correct Answers:

- `step(t-8)*(0.111111111111111*exp( + 7*(t-8)) - 0.111111111111111*exp(-2*(t-8)))`