MATH 552 **Numerical Linear Algebra** **Spring 2006**

Homework Set 3 Due Tuesday, 14 March 2006

1. Problem **4.1** from *Trefethen & Bau*, only parts (a), (c) and (e).
2. Problem **4.3** from *Trefethen & Bau*. You may use MATLAB's *svd* command to generate the singular vectors and values.
3. Problem **4.5** from *Trefethen & Bau*.
4. Problem **5.1** from *Trefethen & Bau*.
5. Problem **5.4** from *Trefethen & Bau*.
6. Find the eigenvalues and a corresponding eigenfunction for the operator $-d^2/dx^2$ applied to functions with homogeneous boundary conditions on $[0,1]$, i.e., solutions of

$$-y'' = \lambda y, \quad y(0) = y(1) = 0.$$

Note that there are a infinite number of them, say $(\lambda_k, y_k(x))$ for $k = 1, 2, \ldots$, with $0 < \lambda_1 < \lambda_2 < \ldots$.

7. Consider the discrete analog of the eigenproblem in the previous problem, with $-d^2/dx^2 \approx -D_+D_- = -D^2$. To this end, choose $N > 0$, let $h = 1/N$ and $x_i = i * h$ for $i = 0, 1, \ldots, N$. Note we now have $N + 1$ grid points with $x_0 = 0$ and $x_N = 1$. So we seek e-pairs which satisfy

$$(-D^2u)_i = \frac{-u_{i-1} + 2u_i - u_{i+1}}{h^2} = \lambda u_i \quad \text{for } i = 1, 2, \ldots, N-1, \quad u_0 = u_N = 0,$$

or in matrix form,

$$\frac{1}{h^2}\begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & -1 \\ & & & -1 & 2 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-2} \\ u_{N-1} \end{pmatrix} = \lambda \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-2} \\ u_{N-1} \end{pmatrix}$$

   (a) Show that (λ_k, u_k) is an e-pair for $k = 1, 2, \ldots, N-1$ where $\lambda_k = 2(1 - \cos k\pi h)/h^2$ and u_k is the vector with components $(u_k)_i = \sin ik\pi h$. Hint: Use trig identities, and note that $\sin 0k\pi h = \sin Nk\pi h = 0$.

   (b) For each $N = 8$ and $N = 16$ plot graphs (using *subplot*) of the continuous eigenfunctions found in the previous problem, and the discrete eigenvectors from (a) for $k = 1, 2, N/2$ and $N - 1$. Thus each plot should have 4 images. Use a linetype for the continuous eigenfunctions and symbols for the discrete eigenvectors. Label and title your graphs.

   (c) In a single plot display the first 32 eigenvalues vs. there number, i.e., $k = 1, 2, \ldots, 32$ of the continuous problem along with the $N - 1$ eigenvalues for each $N = 8, 16$ and 32. For the continuous case plot the eigenvalues using a linetype with a symbol, and in the discrete case just symbols. Label, title, and place a legend on your plot.

Discuss the results.