MATH 552

Spring 2006

Homework Set 2

Due Tuesday, 28 Febuary 2006

- 1. Problem **3.1** from *Trefethen & Bau*.
- 2. Problem **3.2** from *Trefethen & Bau*.
- 3. Problem **3.3** from *Trefethen & Bau*.
- 4. Prove that given a vector norm ||x||, the formula  $||A|| = \sup_{x \neq 0} \frac{||Ax||}{||x||}$  defines a matrix norm for a square matrix A. Recall, this is referred to as the *induced matrix norm*.
- 5. Prove that  $||A||_{\infty} = \max_{i} \sum_{j} |a_{ij}|$ , the maximum absolute row sum of the matrix A.
- 6. Consider the 2-point BVP

$$\begin{cases} -y'' + (4x^2 + 2)y = 2x(1 + 2x^2) \\ y(0) = 1, \ y(1) = 1 + e \end{cases}$$

The exact solution is  $y(x) = x + e^{x^2}$ . Write a MATLAB function M-file to solve the problem using the **4th** order centered compact FD scheme

$$-D_{+}D_{-}\left(1-\frac{h^{2}}{12}c_{i}\right)u_{i}+c_{i}u_{i}=\left(1+\frac{h^{2}}{12}D_{+}D_{-}\right)f_{i}.$$

Use meshsize  $h = 1/2^p$ , where p is a positive integer. Your code should use your M-files **trilu** and **trilu\_solve**. For p = 1 : 4, plot the exact solution (y(x) vs. x) and the numerical solution  $(u_i \text{ vs. } x_i)$ , including the boundary points. The 4 plots should appear separately in one figure, with axes labeled and a title for each indicating p. Investigate **subplot** in MATLAB for how to have multiple plots in a single figure window. For p = 1 : 20 present a table with the following data - column 1: h; column 2:  $||u_h - y_h||_{\infty}$ ; column 3:  $||u_h - y_h||_{\infty}/h^4$ ; column 4: cpu time; column 5: (cpu time)/m, where h = 1/(m + 1). Discuss the trends in each column. Also, compare the accuracy for each h with the results of the 2nd order code from HW #1. How do the computational times compare? Include a copy of your code.