MATH 552 **Numerical Linear Algebra** **Spring 2006**

Homework Set 1 Due Thursday, 16 Febuary 2006

1. Problem **1.4** from *Trefethen & Bau*.
2. Problem **2.2** from *Trefethen & Bau*.
3. Problem **2.3** from *Trefethen & Bau*.
4. Problem **2.4** from *Trefethen & Bau*.
5. Problem **2.5** from *Trefethen & Bau*.
6. Let $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Solve $Ax = b$ (by hand) using the spectral decomposition of A. Show all details.
7. Write a MATLAB function M-file **trilu** to find the LU decomposition as discussed in class, $A = LU$, for the tridiagonal $m \times m$ matrix A,

$$A = \begin{pmatrix} a_1 & c_1 & & & \\ b_2 & a_2 & c_2 & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & c_{m-1} \\ & & & b_m & a_m \end{pmatrix} = \begin{pmatrix} 1 & & & & \\ \beta_2 & 1 & & & \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ & & & \beta_m & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 & c_1 & & & \\ & \alpha_2 & c_2 & & \\ & & \ddots & \ddots & \\ & & & \ddots & c_{m-1} \\ & & & & \alpha_m \end{pmatrix} = LU.$$

   The function should output the two m-vectors α and β, and its first line should read:

   ```
   function [alpha,beta] = trilu(a,b,c)
   ```

   Next, write an M-file function **trilu_solve** to solve $Ax = f$, which takes the vectors α, β, c and f and returns x. Its first line should read:

   ```
   function x = trilu_solve(alpha,beta,c,f)
   ```

   Test your code with the 5×5 system with $a_i = 2$, $b_i = -1$, $c_i = -1$, and RHS $f = [1, 0, 0, 0, 1]^T$. The exact solution is clearly $x = [1, 1, 1, 1, 1]^T$. Use MATLAB's **diary** command to save your MATLAB session output showing that your code works properly. Include a copy of both codes.
8. Consider the 2-point BVP

$$\begin{cases} -y'' + (4x^2 + 2)y = 2x(1 + 2x^2) \\ y(0) = 1,\ y(1) = 1 + e \end{cases}$$

   Show $y(x) = x + e^{x^2}$ is the exact solution. Write a MATLAB function M-file to solve the problem using the 2nd order centered FD scheme we discussed in class, $-D_+D_-u_i + c_iu_i = f_i$. Use meshsize $h = 1/2^p$, where p is a positive integer. Your code should use your M-files **trilu** and **trilu_solve**. For $p = 1 : 4$, plot the exact solution ($y(x)$ vs. x) and the numerical solution (u_i vs. x_i), including the boundary points. The 4 plots should appear separately in one figure, with axes labeled and a title for each indicating p. Investigate **subplot** in MATLAB for how to have multiple plots in a single figure

window. For $p = 1 : 20$ present a table with the following data - column 1: h; column 2: $\|u_h - y_h\|_\infty$; column 3: $\|u_h - y_h\|_\infty / h^2$; column 4: cpu time; column 5: (cpu time)$/m$, where $h = 1/(m+1)$. Discuss the trends in each column. Include a copy of your code.

Hints:

- Debug your code using small values of p, say $p \leq 5$.
- Type **help disp** and **help diary** to learn how to easily display the table results if the table is stored in a matrix, and also how to save MATLAB's output to a file.
- Type **help print** to learn how to save/print the image in a figure window.
- Type **help tic**, and **help toc** to learn how to find the cpu time.
- Try different output formats, e.g. **format long**, **format short e**, **format short g**.