Math 552 **Scientific Computing II** **Spring 2020**

Homework Set 3 Due Friday, 6 March 2020

HW guidelines: You may work together but each individual must submit a separate homework, which must be neatly written or typed, explanations clear and concise, the problems should appear in their given order, m-file code included when requested, all plots created using MATLAB and attached in their proper place, and use the *diary* command to save and print your numerical results computed using MATLAB. You need not include an entire page for just a few MATLAB commands and output, instead cutting and pasting/taping the snippet in your homework in the relevant place.

1. Consider the 2-point BVP

$$\begin{cases} -u'' + (4x^2+2)u = 2x(1+2x^2) \\ u(0) = 1,\ u(1) = 1+e \end{cases}$$

   (a) Show $u(x) = x + e^{x^2}$ is the exact solution.

   (b) Write a MATLAB function M-file to solve the problem using the second order centered FD scheme we discussed in class, $-D^2 v_i + \sigma_i v_i = f_i$. Your code should use your M-files **trilu** and **trilu_solve**. Include a copy of your code.

   (c) For mesh sizes $h = (1/2)^p$, $p = 1:4$, plot the exact solution ($u(x)$ vs. x) and the numerical solution (v_i vs. x_i), including the boundary points. The 4 plots should appear separately in one figure, with axes labeled and a title for each indicating p. Investigate **subplot** in MATLAB for how to have multiple plots in a single figure window.

   (d) For mesh sizes $h = (1/2)^p$, $p = 1:10$ present a table with the following data - column 1: h; column 2: $\|u_h - v_h\|_\infty$; column 3: $\|u_h - v_h\|_\infty / h^2$, where $h = 1/n$. Discuss the trends in each column.

2. Find the eigenpairs $(\lambda, u(x))$ of the Laplacian with homogeneous Dirichlet BCs on the interval $[0,1]$, i.e.

$$\begin{cases} u'' = \lambda u \\ u(0) = u(1) = 0 \end{cases}$$

3. Consider the linear operator $L = d/dx$ on the vector space $\mathcal{P}_2$, the set of all polynomials of degree less than or equal to 2. For each of the bases S below find the matrix A representation of L, i.e. $A = [L]_S$. Then for each find the eigenvalues of A.

   (a) $S_1 = \{1, x, x^2\}$

   (b) $S_2 = \{2, x+1, x^2-1\}$

   What is the relationship between the eigenvalues of A_{S_1} and A_{S_2}?

4. Given a real vector $v = (v_1, \ldots, v_{N-1})^T$ the *Discrete Sine Transform* of v is given by $\hat{v} = P^{-1}v$, where P is an $(N-1) \times (N-1)$ matrix with $P_{i,j} = (2/\sqrt{2N}) \sin(ij\pi/N)$ for $i, j = 1, 2, \ldots, N-1$.

   Show that (a) $P = P^T$ and (b) $P^{-1} = P$. (Note - the sums are most easily computed by writing the sines in terms of complex exponentials.)