Math 551 **Introduction to Scientific Computing** **Spring 2021**

Homework Set 7 Due 9PM on Wednesday, April 21

1. Consider the matrix A and vector x:

$$A = \begin{bmatrix} -1 & 3 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}, \qquad x = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

   (a) Find $\|x\|_2$ and $\|Ax\|_2$.

   (b) Find $\|A\|_1$ and $\|A\|_\infty$.

   (c) Find the eigenvalues of A.

2. Consider solving $Ax = b$, where $A \in \mathbb{R}^{3x3}$, $x \in \mathbb{R}^3$, and $b = [0,\ 1,\ -1]^T$.

   You call an LU decomposition routine that employs partial pivoting with A as an input, which returns L and U stored in the original memory space of A, and the pivot vector p as

$$A = \begin{bmatrix} 1/2 & -1 & 2 \\ 1/2 & -1/2 & 5/2 \\ 2 & 1 & -1 \end{bmatrix} \qquad \text{and} \qquad p = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

   Using these, compute the solution x.

3. Consider an invertible matrix $A \in \mathbb{R}^{nxn}$. For some odd reason (maybe it is a question on a hw) you want to find A^{-1}. Note that $AA^{-1} = I$, which is the same as saying that $A * col_k(A^{-1}) = e_k$, the k-th column of the identity.

   So you execute the following algorithm:

   - Use GE with partial pivoting to find the LU decomposition.
   - Use you LU decomposition to solve $A * col_k(A^{-1}) = e_k$, $1 \leq k \leq n$, thus computing each column of A^{-1} one at a time.

   If $O(Cn^p)$ is the **total** operation count of this algorithm for finding A^{-1}, what are C and p?