Math 551 **Introduction to Scientific Computing** **Spring 2021**

Homework Set 2 Due Thursday, 9PM EST, 25th February

1. Given a function $f(x)$, use Taylor approximations to derive a second order approximation to $f''(x_0)$ is given by

$$f''(x_0) = af(x_0 - h) + bf(x_0) + cf(x_0 + h) + O(h^2)\,.$$

(a) What is the precise form of the error term? (b) Using the formula approximate $f''(1)$ where $f(x) = e^x$ for $h = 1/(2^p)$ for $p = 1:15$. Form a table with columns giving h, the approximation, absolute error and absolute error divided by h^2. For each indicate to which values they are converging. (c) Finally, verify that the last column appears to be converging to a value derived using the error term.

2. Using the quadratic formula, find the two roots of $x^2 - 80x + 2 = 0$. Perform all calculations using rounded 5 decimal digit arithmetic (i.e. $\beta = 10$, $n = 5$), However, the 'standard' formula leads to catastrophic cancellation in the case of one of the roots. Explain why, and then reformulate the formula for this root, and use your result to compute this root.

3. Consider evaluating the integrals

$$y_n = \int_0^1 \frac{x^n}{x + 10}\, dx$$

for $n = 1, 2, \ldots, 30$.

   (a) Show that $y_n + 10y_{n-1} = 1/n$.

   (b) Show that $y_0 = \log 11 - \log 10$ and then use it with the recursion

$$y_n = \frac{1}{n} - 10y_{n-1}$$

   to numerically generate y_1 through y_{30}. Be sure to use y_0 in the form above, **not** $\log 11/10$.

   (c) Show for $n \geq 0$ that $0 \leq y_n \leq 1$, and discuss the results in (b) in light of this.

4. Find $P_2(x)$ for $f(x) = e^x \sin x$ expanded about $x_0 = 0$. Then

   (a) Find a bound on the error $|f(x) - P_2(x)|$ in using P_2 to approximate f on $[-1, 1]$.

   (b) Find the **true** maximum error to at least 5 decimal places. How do they compare? Plot the absolute value of the error between P_2 and f over $[-1, 1]$. Be sure to label the axes, and put a *title* on the plot. Include a copy of your graph.