Math 551 4/29

- HW #8 due @9PM on 4/30
- HW #9 due @5PM on 5/12

Office Hour 7PM - 8PM 4/29

## Iterative Methods

$A = L + D + U \quad (= N - P)$

**Jacobi:**

$$x^{(k+1)} = -D^{-1}(L+U)x^{(k)} + D^{-1}b$$
$$= M_J x^{(k)} + \tilde{b}$$

**G-S:**

$$x^{(k+1)} = -(L+D)^{-1}U x^{(k)} + (L+D)^{-1}b$$
$$= M_{GS} x^{(k)} + \tilde{b}$$

**SOR:** $\omega$ - parameter (in general $\omega > 1$)

$$x^{(k+1)} = (\omega L + D)^{-1}\left[(1-\omega)D - \omega U\right]x^{(k)} + (\omega L + D)^{-1}\omega b$$
$$= M_\omega x^{(k)} + \tilde{b}$$

**Note:** $\omega = 1$ then SOR = G-S

$$x^{(k+1)} = M x^{(k)} + \tilde{b}$$

converges if $\lim_{k\to\infty} x^{(k)} = x$

Thm: If $\|M\| < 1 \Rightarrow$ method converges for all $x^{(0)}$

Thm: The method converges iff $\rho(M) < 1$

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Thm (D. Young) If $A$ is symmetric positive definite (+ 1 more condition Tuesday) then

$$\omega^* = \frac{2}{1+\sqrt{1-(\rho(M_J))^2}}$$

is the OPTIMAL with

$$\omega^* - 1 = \rho(M_{\omega^*}) \leq \rho(M_\omega) < 1$$

$0 < \omega < 2$