Math 551 4/1

- HW #6 due 4/6
- HW #7 assigned 4/6
- Read over 7.1, $\boxed{7.2,\ 7.3}$
- GE/LU is a Direct method

$Ax = b \quad O(\frac{2}{3}n^3)$

Iterative methods

$x^{(0)}$ - initial guess

$x^{(k)} \xrightarrow{?} x^{(k+1)}$

$\lim_{k \to \infty} x^{(k)} = x$?

Partial Pivoting STABILIZES GE

$$A^{(k)} = \begin{bmatrix} \ddots & & \\ & & \begin{bmatrix} a_{k,k}^{(k)} \\ a_{k+1,k}^{(k)} \\ \vdots \\ a_{n,k}^{(k)} \end{bmatrix} \\ 0 & & \end{bmatrix}$$

$R_k \leftrightarrow R_i$ guarantees $|m| \le 1$

Find i such that

$$|a_{i,k}^{(k)}| = \max\left\{ |a_{k,k}^{(k)}|, |a_{k+1,k}^{(k)}|, \ldots, |a_{n,k}^{(k)}| \right\}$$

Ex: $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 4 & 2 \\ 4 & 1 & 3 \end{bmatrix}$ $\max\{|1|, |-4|, |4|\} = 4$

$n=3$

$\boxed{n = 10^8}$

$R_1 \longleftrightarrow R_3$

① store [1 3 5] in temp

② copy [4 1 3] to R_1

③ copy temp to R_3

A LOT of MEMORY MOVEMENT!

Pivot Vector:

Initialize $P = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{array}{l} a_{i,j} \to a_{p(i),j} \\ b_i \to b_{p(i)} \end{array}$

$R_1 \leftrightarrow R_3$ as $p(1) \leftrightarrow p(3)$

$p(1) \to temp$
$p(3) \to p(1)$
$tem \to p(3)$

$\Rightarrow p = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

where is row 3?

$p(3) = 1$, so it is physically in row 1, but we indirectly address it.

Implementation of the Pivot Vector

① initialize $p = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix}$

② Find the pivot row i such that

$$|a^{(k)}_{p(i),k}| = \max\left\{ |a^{(k)}_{p(k),k}|, |a^{(k)}_{p(k+1),k}|, \ldots |a^{(k)}_{p(n),k}| \right\}$$

③ $p(i) \longleftrightarrow p(k)$ accomplishes the row interchange indirectly!