

Math 551

4/1

- HW #6 due 4/6
- HW #7 assigned 4/6

• Read over 7.1, 7.2, 7.3

• GE/LU is a
Direct method

$$Ax=b \quad O\left(\frac{2}{3}n^3\right)$$

Iterative methods

$x^{(0)}$ - initial guess

$$x^{(k)} \xrightarrow{?} x^{(k+1)}$$

$$\lim_{k \rightarrow \infty} x^{(k)} = x_i?$$

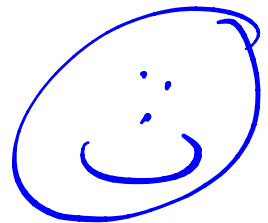
Partial Pivoting

STABILIZES GE.

$$A^{(k)} = \left[\begin{array}{c} \vdots \\ 0 \\ \vdots \end{array} \left[\begin{array}{c} a_{k,k}^{(k)} \\ a_{k+1,k}^{(k)} \\ \vdots \\ a_{n,k}^{(k)} \end{array} \right] \right]$$

$R_k \Leftrightarrow R_i$
guarantees

$$|m| \leq 1$$



Find i such that

$$|a_{i,k}^{(k)}| = \max \left\{ |a_{k,k}^{(k)}|, |a_{k+1,k}^{(k)}|, \dots, |a_{n,k}^{(k)}| \right\}$$

Ex: $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 4 & 2 \\ 4 & 1 & 3 \end{bmatrix}$ $\max\{|1|, |1-2|, |4|\} = 4$

$n=3$

$n = 10^8$

$R_1 \leftrightarrow R_3$

- ① store $[1 \ 3 \ 5]$ in temp
- ② copy $[4 \ 1 \ 3]$ to R_1
- ③ copy temp to R_3



A LOT OF MEMORY MOVEMENT!

Pivot Vector:

Initialize $P = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow a_{ij} \rightarrow a_{p(i),j}$
 $b_j \rightarrow b_{p(i)}$

$R_1 \leftrightarrow R_3$ as $P(1) \leftrightarrow P(3)$

$p(1) \rightarrow \text{temp}$
 $p(3) \rightarrow p(1)$
 $\text{temp} \rightarrow p(3)$

$$\Rightarrow P = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

Where is row 3?

$p(3) = 1$, so it
is physically in row 1,
but we indirectly address it.

Implementation of the Pivot Vector

- ① initialize $p = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix}$
- ② Find the pivot row i such that
$$|a_{p(i),k}^{(k)}| = \max \{ |a_{p(k),k}^{(k)}|, |a_{p(k+1),k}^{(k)}|, \dots, |a_{p(n),k}^{(k)}| \}$$
- ③ $p(i) \leftrightarrow p(k)$ accomplishes the row interchange indirectly!