HW 8 due 4/30 @ 9PM

1. $A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$, $x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

a) Find M_J.

b) With $x^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, compute IN EXACT ARITHMETIC $x^{(1)}$, $x^{(2)}$ and $x^{(3)}$

c) Find the e-vals and corresponding e-vectors of M_J

d) Using the results in (c) prove $\lim_{k \to \infty} x^{(k)} = x$

2. Same as 1 except Gauss-Seidel instead of Jacobi, i.e., M_{GS}

3. Consider the iteration

$$x^{(k+1)} = b + \alpha \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} x^{(k)} \quad \text{for } k = 0, 1, 2 \ldots$$

where $\alpha \in \mathbb{R}$. For what values of α does the iteration converge for any choice of $x^{(0)}$?