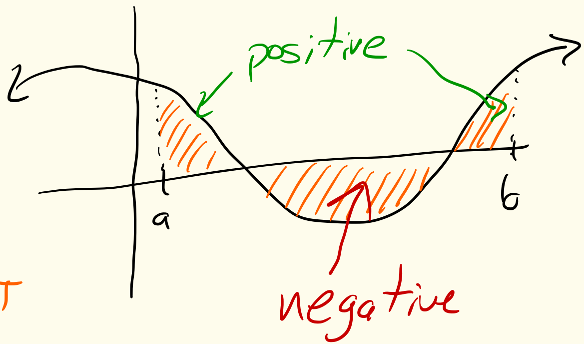
Integral Calculus

$y = f(x)$



$$\text{"Area"} = \overbrace{\int_a^b f(x)\,dx}^{\text{DEFINITE INTEGRAL}}$$

$$\overset{FTC}{=} \boxed{F(b) - F(a)}$$ EXACT ANS

where $F'(x) = f(x)$, i.e. $F(x)$ is a $\begin{cases} \text{indefinite integral} \\ \text{anti-derivative} \end{cases}$

Problem: Finding $F(x)$ can be difficult!!

BRING YOUR CALCULATOR to class!

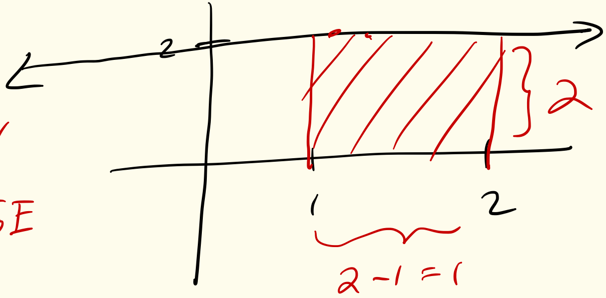
6.1: Average Value of a Function

Ex: average or mean of $\{1, 2, 4\}$ avg $= \frac{1+2+4}{3} = \frac{7}{3}$

DISCRETE

what about if the data is represented by a function?

Ex: $f(x) = 2$



A constant function should have the SAME AVERAGE over any interval!

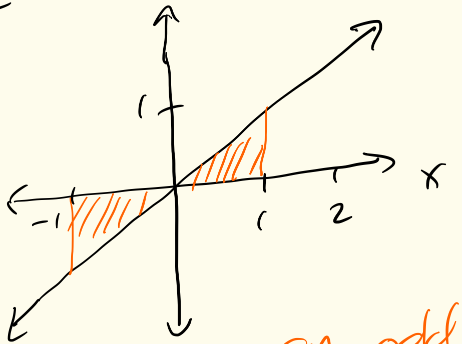
$$2 = \underset{\text{over } [1,2]}{\text{avg } f(x)} = \frac{\text{area over the interval}}{\text{length of the interval}}$$

or $\boxed{\text{avg } f(x) \text{ over } [a,b] = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{\text{"Area"}}{\text{length of interval}}}$

Ex: $f(x) = 2$ over $[1,2]$: $\text{avg} = \frac{1}{2-1}\int_1^2 2\,dx$

$$= \frac{1}{1}\cdot 2 = \boxed{2}$$

Ex: $f(x) = x$



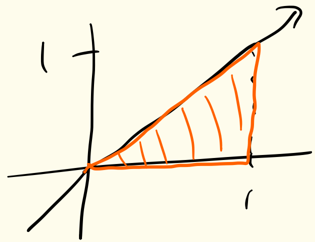
avg over $[-1,1]$ $= \frac{1}{1-(-1)}\int_{-1}^{1} x\,dx$

$a = -1$
$b = 1$

$$= \frac{1}{2}\cdot 0 = \boxed{0}$$

makes sense since $f(x)$ is an odd function over $[-1,1]$.

Ex: $f(x) = x$ over $[0,1]$



$$\text{avg} = \frac{1}{1-0}\int_0^1 x\,dx$$

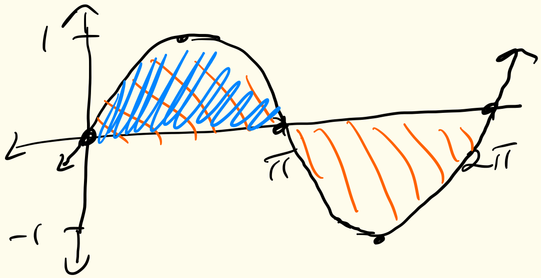
$$= \frac{1}{1}\boxed{\underbrace{\left(\frac{1}{2}\cdot 1\cdot 1\right)}_{\text{area of triangle}}} = 1\cdot\frac{1}{2} = \boxed{\frac{1}{2}}$$

Using an anti-derivative?

$$\int_0^1 x\,dx \overset{FTC}{=} F(1) - F(0) = \left(\frac{1^2}{2} - \frac{0^2}{2}\right) = \frac{1}{2} - 0 = \boxed{\frac{1}{2}}$$

$$f(x) = x \implies F(x) = \frac{x^2}{2} + C$$

Ex: Find avg of $f(x) = \sin x$ over $[0, 2\pi]$



$$\text{avg} = \frac{1}{2\pi - 0}\int_0^{2\pi} \sin x \, dx$$

$$= \frac{1}{2\pi}\boxed{\int_0^{2\pi} \sin x \, dx}$$

$$= \frac{1}{2\pi}\cdot 0 = 0$$

Ex: $f(x) = \sin x$ over $[\overset{a}{\overset{\shortparallel}{0}}, \overset{b}{\overset{\shortparallel}{\pi}}]$

$$\text{avg} = \frac{1}{\pi - 0}\boxed{\int_0^{\pi} \sin x \, dx} = \frac{1}{\pi}\cdot \boxed{2} = \frac{2}{\pi} = 0.63662\ldots$$

$$\approx 0.63662$$

Ex: Town population is given by $P(t) = 570 \cdot (1.037)^t$ where P is measured in 1000's and t in years since 2000.

what is the avg population from 2010 to 2020?

ans: $P(0) = 570 \cdot (1.037)^0 = 570 \cdot 1 = 570$

$P(10) = 570 \cdot (1.037)^{10} \approx 819.714$

$$\text{avg} = \frac{1}{20-10} \int_{10}^{20} P(t)\,dt = \frac{1}{10} \int_{10}^{20} 570 \cdot (1.037)^t \, dt$$

$$= \frac{1}{10} \cdot 9884.22\ldots \approx \boxed{988.422}$$