Revisions to second printing: *Reflection Groups and Coxeter Groups*

5 The last two lines should read: "number of signs. This semidirect product is also a reflection group, generated by the reflections in $\mathcal{S}_n$ and the reflections $\varepsilon_i + \varepsilon_j \mapsto -(\varepsilon_i + \varepsilon_j)$, $i \neq j$."

8 Conclude the statement of (a) in the Theorem with "(denoted Φ^+ if Δ is understood)."

9 Add to Exercise 2: "When $W = \mathcal{D}_m$, the angle between the two simple roots is $\pi - (\pi/m)$."

21 Omit "inductively" in Exercise 1.

25 In line 16, insert after "More precisely,": "the following proposition shows that". In line −11, read "$\alpha \in \Delta_I$" rather than "$\alpha \in I$".

30 In line 5 of proof of Proposition 2.1, read: "the angle θ between them is $\pi - \pi/m(\alpha, \beta)$ (1.3, Exercise 2). Since ..."

31 In last full paragraph, begin the second sentence with "The (leading) **principal minors** ...", and replace the third sentence with: "Then A is positive definite if and only if all its principal minors are positive."

35 In line −23, read "...into nonempty sets I, J such that $a_{ij} = 0 = a_{ji}$."

35 Remove the $\sum$ at the beginning of line −6.

36 In line 4, replace "each i" by "$i \in I$".

37 Replace all occurrences of n by k in steps (3), (5), (12), (13).

42 In the last line, replace c by c_i.

43 In line −7, replace *odd* by *even*.

48 In next-to-last line, replace D_3 by D_6.

48 Expand last sentence in Notes to a new paragraph, as follows: Sekiguchi-Yano [2] show how to embed H_3 in D_6. Lusztig [3, Rem. 3.9(b)] obtains an embedding of H_4 in E_8 as a byproduct of his further exploration of Hecke algebras and W-graphs (beyond Kazhdan–Lusztig [1]). Shcherbak [1] gives a unified treatment for H_2 (dihedral of order 10),

H_3, H_4. His context is far from Lusztig's (whose work he does not mention), but he does cite Sekiguchi-Yano. Although these papers motivate the embeddings in different ways, with divergent methods of proof and varying amounts of detail, all begin with a homomorphism from a non-crystallographic Coxeter group into a crystallographic group having twice the rank. For example, each vertex in the Coxeter graph of H_4 is assigned to a non-connected pair of vertices in the graph of E_8 (so the corresponding product of two reflections again has order 2) via a sort of "folding" of the latter graph. By the Coxeter relations, this defines a homomorphism of the first group into the second (which is not obviously injective).

56 Following the displayed equation (16), read: "with r_i homogeneous and $\deg r_i > 0$."

65 In the statement of Theorem 3.11, replace $\mathrm{GL}(V)$ with $\mathrm{O}(V)$.

71 In part (a) of the Lemma, replace $\chi(1_H^G)$ by $\chi \cdot 1_H^G$.

75 In line -4, read: "If ζ is the primitive hth root of unity $\exp(2\pi i/h)$, these ..."

76 In line 6, read: "In particular, a Coxeter element should have ..." In the first line of the proof of the Lemma, read: "Suppose $w := s_1 \cdots s_n$ fixes some λ."

76 Add a line to the Exercise: "What can be said about exponents?"

78 In lines -11 and -10, change w^t to w^m.

78 Expand the sentence starting on line -10 as follows: "It follows that w has order precisely h on P. Moreover, the closure of $P \cap C$ is the usual fundamental domain of the dihedral group generated by y and z on P, so w acts as a rotation through $2\pi/h$." (This addition requires tightening of the spacing at the top of page 78.)

81 In line -18, read: "In turn, when $i > 1$,"

81 Rewrite line -14: "This (and a similar calculation when $i = 1$, using $m_1 = 1$) forces"

82 The first sentence in Exercise 2 should read: "If h is even and w is the Coxeter element in 3.17, set $z := w^{h/2}$."

108 The exercise in 5.2 should be moved to section 5.8, where the Exchange Condition can be used for the “only if” part.

113 Replace the sentence starting on line 1 with two sentences: “If $m = 2k{+}1$ is odd and $\ell(v_I) = 2k$, then $v_I(\alpha_s) = \alpha_{s'}$. Otherwise the rotation part of v_I moves α_s through at most $\pi - 2\pi/m$, still within the ...”

120 Rewrite lines 5–10 of the proof of Theorem 5.10, replacing four occurrences of w' by w'', as follows.

“argument can be iterated: If in turn $w'' \to w'$, with $w' = w''t'$, apply the Strong Exchange Condition to the pair $t', w' = s_1 \cdots \hat{s}_i \cdots s_r$ (which is not required to be a reduced expression!) to obtain

$$w'' = w't' = s_1 \cdots \hat{s}_i \cdots \hat{s}_j \cdots s_r$$

or else

$$w'' = s_1 \cdots \hat{s}_j \cdots \hat{s}_i \cdots s_r.\text{”}$$

141 Perhaps add a comment after the text on this page: ”It is interesting to compare the following tables with the more elaborate tables in a 2010 paper by L. Carbone et al. in *J. Phys. A: Math. Theor.* **43** on hyperbolic Dynkin diagrams and related matters.”

151 In lines 6–8, the Exchange Condition is not actually needed. Read instead: “The hypothesis $sw < w$ implies that w has a reduced expression $w = s_1 \cdots s_r$ with $s_1 = s$: here $s_2 \cdots s_r$ can be any reduced expression for sw.”

161 In line −13, replace −1 by $-q$.

162 In the summation on line 1, replace C_v by C_z.

165 In line −9, replace ε_x by ε_w.

172 Rewrite the last paragraph of 8.1 as follows: “More generally, one can study the *Conjugacy Problem*: given $w, w' \in W$, decide whether or not they are conjugate. Appel–Schupp [1] solved the problem when W is ‘extra-large’ (all $m(s, s') \geq 4$ when $s \neq s'$). The work of Moussong [1] made it possible to solve the problem for arbitrary Coxeter groups, as explained and refined by Krammer [1].”

180 In third paragraph of 8.10, read Frame [1][2] instead of Frame [1]. (A reference Frame [2] must also be added.)

188 In the reference Conway et al., “T.R. Curtis” should be “R.T. Curtis”.

195 In item 6 under G. Lusztig, remove period after “Pure”.

(February 2015)

Updated references

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