Conjugacy Classes in Semisimple Algebraic Groups (Revisions)

Subtract 4 from each page number in the Index. (This production error seems to be corrected in the paperback reprint.)

7 In line 3 replace $M_{\mu-k\alpha}$ by $M_{\mu+k\alpha}$.

8 In line -11 read $\mathfrak{sl}(r+1,K)$.

8 In line -1, replace "codimension 2" by "codimension 4".

9 In line 5, replace "dimension $2r$" by "codimension $2r$".

9 In line 8, just before "The center of $\mathfrak{g}$ is", insert: "Let $p = 2$."

18 The second sentence following Proposition 1.9 should conclude: "...the orbit map $G \to \mathrm{Cl}(x)$ induces an isomorphism $G/C_G(x) \cong \mathrm{Cl}(x)$."

19 Replace the period at the end of line -2 with a colon.

20 In line -4 read "semisimple".

22 At end of line -8 read "elements of G". At the beginning of line -2 read: "Obviously $C \supset C_G(X) \cap C_G(x) \ldots$

29 In the final paragraph, delete the sentence "For any abelian group A ...". Rewrite the following sentence: "We want to show that $K^\times$ is isomorphic to the subgroup $(\mathbb{Q}/\mathbb{Z})_{p'}$ of $\mathbb{Q}/\mathbb{Z}$ consisting of elements of order prime to p. Clearly $(\mathbb{Q}/\mathbb{Z})_{p'} \cong \mathbb{Q}_{p'}/\mathbb{Z}$, where $\mathbb{Q}_{p'}$ consists of rational numbers representable as fractions a/b with $p \nmid b$."

37 In line 12 read "$\varepsilon_i - \varepsilon_j \in \Phi \cap \overline{A} \ldots$" In lines 15–17, read "If precisely two such components occur, $\overline{\Psi} - \Psi$ yields a single nonzero coset in the Klein 4-group $\overline{A}/A$, which is not annihilated by some character (of order 2, prime to p)."

In line -10, read "*Let H be a connected*".

38 Add a paragraph: "While the methods used by Deriziotis were limited to prime characteristic, a treatment of these ideas over $\mathbb{C}$ (or apparently over any algebraically closed field of characteristic 0) was given by Lusztig in *Classification of unipotent representations of simple p-adic groups*, Internat. Math. Res. Notices **1995**, no. 11, 511–589: see 5.5.

This relies heavily on Lemma 5.4 as well as on the algorithm of Borel and de Siebenthal. A uniform proof in all (good) characteristics would still be desirable."

45 The last symbol on line 11 should be g_0.

57 In line 3, read "$\lambda \in P$". Replace lines 6–7 by: "Moreover, one can totally order the vector space E in a way compatible with the positive system in Φ, so that $\mu > 0$ whenever $\langle \lambda, \mu \rangle > 0$. (Use a lexicographic ordering with λ as first basis vector.)" Delete line 14: "Unless α ..."

60 In line -7, read $\mathrm{F}_4, \mathrm{E}_7$ in place of E_7. The error here in the computations of Lou [112] was corrected by Liebeck and Seitz in their 2012 book added to the list below.

61 In line 11, replace "which in turn" by "whose identity component".

64 In line -12, replace "a positive root in Φ'" by "a positive root".

65 Begin line -11 with "The restriction to X of this map ..." In line -2 read "$\alpha_k(t) = c_k$".

66 In line 9, replace "lies in U_i" by "lies in V_i".

67 In line -5 read: "the regular unipotent elements in G' are dense in the unipotent variety of G'".

68 Replace the sentence beginning "Now we get ..." on line 6 with: "Now use the method of 0.15 to construct a closed subset of $G/C_G(s) \times G$ mapping onto $F - R$, by taking pairs $(gC_G(s), x)$ for which $g^{-1}xg \in I$." Replace line 9 by: "Most of the fibres of π consist of a single class (a regular semisimple class)."

69 In line 8, delete extra "that".

71 In last line of (1), read $\chi_1(g)$ in place of $\chi(g)$.

72 In line 25, replace $\chi(x)$ by $\chi_1(x)$.

73 In line 4, read: "The differentials $d\chi_i$ are independent at x ..." In line 15, reference should be Steinberg [**185**, (2)].

74 In lines 8 and 12, replace $\tau_\pm$ by $\tau_i^\pm$. Expand the proof in the final paragraph of 4.21, giving more explicit details.

79 In line 4, omit the extra "an". Read "$\{\alpha \in \Phi^+ \mid w(\alpha) < 0\}$." at the end of line 15. In line -6 replace "proposition" by "theorem".

80 Replace the first sentence of Theorem 5.3 (iii) by: "Each element $u \in C$ lies in only finitely many conjugates of V."

81 Line 5 should begin: "$\dim \mathrm{Cl}_P(u) = \dim V$."

81 Replace the first two sentences in the second paragraph of the proof by: "Theorem 5.2 shows that there exist elements of $\mathrm{Cl}(V)$ lying in only finitely many conjugates of V, which (by the proof of Corollary 5.2) form an open subset of $\mathrm{Cl}(V)$. This set is clearly a union of conjugacy classes, so it includes the chosen open class $\mathrm{Cl}(u)$.

91 Add to 5.10 a discussion of Borho's approach to the proof, with added citation.

93 In line 8 read "G is a connected semisimple group of rank r."

96 In the last line, replace h_i by $h^{(i-1)}$.

99 In line 9 read "the fibres gB of ..." In line -13 replace "$v =$" by "$v \in$".

101 In line -3 follow the displayed set by "$\to G/B^{(w)}$".

102 Replace the display in line 13 by

$$\dim C_G(u) + (\dim(C \cap U^{(w)}) - \dim U^{(w)}) = r + 2 \dim \mathfrak{B}_u.$$

103 In line 24 replace $\pm\alpha_j$ by α_j. In line 27 replace "Each of the root systems" by "The Weyl group of each root system".

106 In line -6 replace $q \in Q_\beta$ by $q \in P_\beta$.

110 In line 23 replace V/F_{n-i} by V/V_{n-i}.

111 In line -7 replace "case-by-work" with "case-by-case work".

112 In (ii) of the lemma, replace W/W' by $W' \backslash W$.

113 In line 4, replace W/W' by $W' \backslash W$.

116 Reword the Remark at the end of section 6.19 as follows. End the first sentence at: [170, §5]. Then write: "For example, $\mathfrak{g}$ may be obtainable by base change from a Chevalley $\mathbb{Z}$-form as happens when G is simply connected. Otherwise $\mathfrak{g}$ might vary: see 0.13."

118 In line 1 of 6.21, read "Theorem 6.20". In lines -14 to -13, replace "a 1-dimensional additive subgroup" by "an r-dimensional affine space".

127 Replace the sentence on lines 3–6 by: "This equals $d_0 + d_1$, since each irreducible $\mathfrak{a}$-summand of $\mathfrak{g}$ has a 1-dimensional intersection with $\mathfrak{g}_0$ or $\mathfrak{g}_1$ (but not both). Among these are $d_0 - d_2$ trivial $\mathfrak{a}$-summands. To summarize:"

128 Replace line -6 by

$$\lambda_1{-}1, \lambda_1{-}3, \ldots, -(\lambda_1{-}3), -(\lambda_1{-}1), \ldots, \lambda_d{-}1, \lambda_d{-}3, \ldots, -(\lambda_d{-}3), -(\lambda_d{-}1)$$

In line -9 read $\mathfrak{sl}(2, K)$.

131 In line -2 replace $x_{-\beta}$ by y_β.

133 In lines 17–19, read "... is needed because centralizers need not be connected. We say that a unipotent element u (or its class) is distinguished if the group $C_G(u)^\circ$ is unipotent."

133 In line 26 read "distinguished".

138 In the first line below the table, read "come from".

141 In line -11 replace "fails to be normal" by "has non-normal closure".

148 In line 10 replace $a \equiv bF(c)^{-1}$ by $a \equiv cbF(c)^{-1}$. In the last line read "upper unitriangular group".

149 The first symbol on the first line should be G (not U).

154 In line 14 read "if and only if it lies in". In part (ii) of the theorem, read "in $|C_U(u)/C_U(u)^\circ|$ classes." In line -11 replace q^{m-r} by $q^{m-r}(q-1)^r$.

169 Reword the second sentence of (4) in 9.6: "This map respects the gradings, and its image lies in the fixed point space ..."

176 In line 2, cite the paper by Shi.

188 In the reference [112], the page numbering should be 1144–1146.

190 J.P. Serre, Sém. Bourbaki (1993–94), Exposé 783, Astérisque **227** (1995).

192 D.M. Testerman, J. Algebra **177** (1995), 34–76.

The added parenthetic remark on page 38 originates in questions raised by Roberto Rubio; then Lusztig's former student Eric Sommers pointed out the treatment in characteristic 0 given in Lusztig's 1995 paper.

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Added references

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