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Statis 697G Survival Analysis HW #1
Due on Friday, February 5, 2016

Note: Your computer output is just for references for your answers. You must write down your answer for each question asked (cut and paste is fine, but specify the title for the values presented). If you use computer, please attach your computer program codes at the end.

1. Consider a *k-out-of-n* system containing n independent units with i.i.d. lifetimes $T_1, \ldots, T_n$. Show that the system lifetime has the same distribution as that of $T_{(n-k+1)}$, the $(n-k+1)th$ order statistic of $T_1, \ldots, T_n$.

2. Suppose there are 3 identical units and their lifetimes are independent. The probability hat each unit will be fuctioning at time t is p. Compute the reliability of each of the following syatems at time t.

   (a) 3 units are in series.

   (b) 3 units are in parallel.

   (c) They form a *2-out-of-3* system.

   (d) Graph the above three reliabilities as as functions of p , $0 \leq p \leq 1$. Commend on your observations, e.g., under what situations which one is better than which one.

3. Two components have independent exponential distributions with p.d.f.s

$$\lambda_1 e^{\lambda_1 t} \ \ and \ \ \lambda_2 e^{\lambda_2 t},$$

   respectively, for $t \geq 0$ and zero elsewhere. Suppose a system is a parallel configuration of these components.

   (a) Find the reliability function of the syaytem.

   (b) Find the hazard rate of the system. Plot the graphs on the same frame for the hazard functions for $\lambda_1 = .1$, $\lambda_2 = .9$, and $\lambda_1 = .3$, $\lambda_2 = .7$. Make comments on your graphs.

4. A certain type of transistors is known to have a constant failure rate $\lambda = .04\%/K$ ($1\%/K$ = one percent per 1000 hrs= an expected rate of 1 failure for each 100 units operating 1000 hrs.) (a) What is the MTTF? (b) What is the probability that one of these transistors fails before 15,000 hrs of use? (c) How long do we have to wait to expect 1% failures?

5. An electronic manufacturer uses interconnection wire which has a nominal strength of 11 g (i.e., it takes an average pull force of 11 g to break the wire). If the population with this average strenth is normal with a standard deviation of 1.2 g, find the following:

   (a) The proportion of wires which will survive a pull of 13 g.

   (b) The probability of a wire breaks under a load of 8 g.

   (c) The proportion of wires which will survive a pull of 8.5 g but not a pull of 13.2 g.