MATH 621 COMPLEX ANALYSIS, SAMPLE MIDTERM EXAM/HOMEWORK 5

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Part I. Stein-Shakarchi Chapter 2, pp. 64ff: 8, 9, 10, 12(a), 13. Extra Credit: 14.

For 13) One possible elaboration on the hint, which you may or may not find useful: Write $\mathbb{C}$ as a countable union of compact sets $K_1, K_2, \ldots$. Define for $n \geq 0$, Ω_n to be a certain subset of $\mathbb{C}$ related to the condition given in the problem. Now compare where the Ω_n's sit in relation to the K_ν's keeping in mind basic notions of countability.

Part II.

1. Let
$$f(z) = \cos(\frac{1+z}{1-z}), |z| < 1,$$
be defined on the open unit disc $\mathbb{D}$. Let $Z_f = \{z \in \mathbb{D} \mid f(z) = 0\}$. Determine this set. Does it have any accumulation points?

Note: $\cos(z) = (e^{iz} + e^{-iz})/2$.

2. Suppose γ is the unit circle traversed counterclockwise once.
a) For integers $n \geq 0$, compute
$$\int_\gamma \frac{\cos(z)}{z^{n+1}} dz.$$
Can you do it for negative integers n also?

b) For non-negative integers n, evaluate
$$\int_\gamma \frac{\cos^n(z)}{z^3} dz.$$

Due February 24, 2011.