MATH 621 COMPLEX ANALYSIS, HOMEWORK 1

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Part I. Stein-Shakarchi Chapter 1, pp. 24-25: 1, 3, 7.

Part II.

1. Express the following as $a + bi$ with explicit $a, b \in \mathbb{R}$.
(a) $(1+i)^{100}$
(b) $((i+1)(2-3i)(3-i))^{-1}$
(c) $\sum_{k=0}^{n} i^k$.

2. In each case, sketch the set of points satisfying the given condition. Argue geometrically as much as possible.
(a) $|z+1-2i| > 2$
(b) $\mathrm{Re}(z) \leq \mathrm{Re}(z^2)$
(c) $z\overline{z} + z + \overline{z} < \mathrm{Im}(z)$
(d) $\mathrm{Re}(\frac{z-1}{z+1}) = 0$
(e) $|z+1| \leq |z-1|$
(f) $|z+i| + |z-i| \leq 4$.

3. Express in polar form:
(a) $\sqrt{3} + i$
(b) $i - 1$
(c) $(i-1)^3(\sqrt{3}+i)^2$

4. Suppose $a, b \in \mathbb{R}_{>0}$. Prove that

$$\tan^{-1}\left(\frac{1}{a+b}\right) + \tan^{-1}\left(\frac{b}{a^2+ab+1}\right) = \tan^{-1}(\frac{1}{a}).$$

What happens for arbitrary real a, b?

5. Find all possible numbers representing the root below, and represent them geometrically:
(a) $(-i)^{1/4}$
(b) $(1-i\sqrt{3})^{1/5}$

6. Find the four roots of the equation $z^4 + 4 = 0$ and use them to factor $z^4 + 4$ into two quadratic factors with real coefficients.

7. Express each of the following functions as $f = u + iv$ where u, v are functions from $\mathbb{R}^2$ to $\mathbb{R}$, i.e. compute $u(x, y) = \mathrm{Re}(f(x+iy))$ and $v(x, y) = \mathrm{Im}(f(x+iy))$.
(a) $f(z) = 1/z^2$ (b) $f(z) = (z-i)/(z+i)$.

The due date is Tuesday January 25, 2011.