UMASS AMHERST MATH 411 SECTION 2, FALL 2009, F. HAJIR

SAMPLE QUESTIONS FOR EXAM 2

Here are some sample questions for the final. Check back here in a few days.

1. As usual, the first question will ask you for the definition of various objects. I might ask you for the definition of any object defined in class or in the handouts, so review all of these.

2. Suppose a finite group G of odd order acts on a set X and $|X| = 2$. Prove that G must act trivially on X i.e. $g.x = x$ for all $g \in G$ and all $x \in X$.

3. (a) Write the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 4 & 2 & 3 & 5 & 7 & 1 \end{pmatrix}$$

as a product of disjoint cycles.

(b) Now write σ as a product of transpositions.

4. Let p be a prime number. Suppose a finite group G of order p acts on a finite set X where $|X|$ is divisible by p. Suppose there is at least one fixed point $x \in X$ under the action of G. Show that there are at least p fixed points under the action of G.

5. Prove that if G is a group of order 4, then G is commutative.

6. Suppose G is a group and $\psi : G \twoheadrightarrow S_4$ is a surjective homomorphism with kernel $H = \ker(\psi)$.

(a) Can we conclude that G is finite?

(b) If we are told that H is finite, can we conclude that G is finite?

(c) Can we conclude that G is not commutative?

(d) If H is non-trivial, then can we conclude that G is not isomorphic to S_4?

7. Prove that a group of prime order is simple.

8. Suppose G is a group such that all subgroups of G are normal in G. Show that for each $a, b \in G$ there exists $j \in \mathbb{Z}$ such that $ba = a^j b$. Discuss the uniqueness of j in $\mathbb{Z}$.

9. (a) Suppose G is a group, and H is a normal subgroup of G such that G/H is commutative (i.e. $abH = baH$ for all $a, b \in G$). Show that for all $a, b \in G$, $a^{-1}b^{-1}ab \in H$.

(b) Suppose G is a group and H is a normal subgroup of G such that for all $a, b \in G$, $a^{-1}b^{-1}ab \in H$. Prove that G/H is commutative.

10. Suppose G is a group with subgroups M and N with $|M| = m$ and $|N| = n$. Assume that $\gcd(m, n) = 1$. Prove that $M \cap N = \{e\}$.

11. Suppose $\psi : G \twoheadrightarrow Q$ is a surjective homomorphism. Suppose H is a normal subgroup of G. Prove that $\psi(H)$ is a normal subgroup of Q.

12. Show that if H is a normal subgroup of a group G, then H is the union of a collection of conjugacy classes of G.