UMASS AMHERST MATH 300 FALL '05, F. HAJIR

EXAM 1 REVIEW

The material you should know for Exam 1 is as follows: everything that appears in HW 1-3 and in the readings that were assigned for HW 1-3. The exam will have three parts: 1. Defintions, 2. Short Answer, and 3. Problems. The problems will usually but not always require you to write a cogent, concise and correct proof. Some of the problems will be statements that were already proved in class or are taken directly from homework. But at least some of the problems will require you to prove a statement that has not been presented to you before, or to find a counterexample to a statement.

For the definitions, it is important to be extremely precise. For instance if I ask you define what it means for $f : X \to Y$ to be surjective, the response "f is surjective means that for all $y \in Y$, there exists $x \in X$ such that $f(x) = y$ receives full credit, and "everyting in Y gets hit by somebody in X" receives only partial credit because "gets hit by" is not sufficiently precise. If you say f is surjective means that $f(X) = Y$, you will receive full credit only if you provide the definition $f(X) = \{f(x) \mid x \in X\}$ as well.

The points are distributed approximately as follows: 25% Definitions, 25% Short Answer, and 50% Problems. For Extra Credit: 1 point total for the trivial credit, 5 points for the non-trivial credit, and 10 points for the highly non-trivial credit.

You may wish to give yourself 1.5 hours and take this exam in a quiet room without notes under the time constraint (or not, this is just a suggestion; it may be a good suggestion for some students and not so good for others). The actual exam will be somewhat similar but not identical to this one in length and in the variety of the problems.

Here begineth the sample exam.

Sample Exam 1

1. Definitions

A set is

A set X is finite means that

A bijection from X to Y is

If $f : X \to Y$ and $g : Y \to Z$ are maps, then the composite map $g \circ f$ has source .... and target .... and is defined by

A set X is equal to a set Y means that

A set X is a subset of a set Y means that

The intersection of X and Y is defined by

The power set $\mathcal{P}(X)$ is

A partition of a set X is

The direct or Cartesian product of X and Y is the set $X \times Y =$

A map $f : X \to Y$ is invertible if

The nth triangular number, or bowling number, is
We say that two statements P and Q are equivalent if

2. Short Answer

The converse of $-P \Rightarrow -Q$ is
The negation of $(P \wedge Q)$ is $-(P \wedge Q) =$..........
Let R be the statement: Whenever it rains, my car gets wet. State the negation of R.
Determine whether $(-P \vee -Q) \Leftrightarrow -(P \wedge Q)$ is a tautology.
Write down two sets, X and Y, say, which are equivalent, but not equal.
Construct the truth table for $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$ and determine whether it is equivalent to $(Q \Rightarrow P)$.
Give a bijection $[0, 1] \to [0, 1]$ other than the identity map.
For each statement below, Indicate whether it is True or False.
If X is equal to Y, then $X \subseteq Y$ and $Y \subseteq X$.
If $f : X \to Y$ is surjective and $g : X \to Y$ is injective then $g \circ f$ is bijective.
If X and Y are equivalent sets, then X is finite if and only if Y is finite.
If X is an infinite set, then every subset of X is equivalent to X.
If X is a set, then $\{\} \subseteq X$.
If $X \subseteq Y$ and $Y \subsetneq Z$, then $X \subsetneq Z$.
If X is finite, then $\mathcal{P}(X)$ is also finite.
The set $\mathbb{N} = \{1, 2, 3, \ldots\}$ is equivalent to the set $\{y \in \mathbb{Z} | y \geq 1984\}$.
If $f : X \to Y$ is surjective and $g : Y \to Z$ is surjective, then $g \circ f$ is surjective.

3. Problems

1. Prove that if X is a set, then there is an injection $f : X \hookrightarrow \mathcal{P}(X)$.
2. Consider the following sequence of statements: $S_1 : 2 = 1 \cdot 2$, $S_2 : 2 + 4 = 2 \cdot 3$, $S_3 : 2 + 4 + 6 = 3 \cdot 4$, $S_4 : 2 + 4 + 6 + 8 = 4 \cdot 5$. Using inductive reasoning, write down your expectation for statement S_n for an arbitrary positive integer n.
3. Suppose A, B, C are sets. (a) Show that $C \setminus (A \cup B) \subseteq C \setminus A$.
(b) Give explicity three sets A, B, C such that $C \setminus (A \cup B)$ is a proper subset of $C \setminus A$.
4. Prove or Disprove: Given maps $f : X \to Y$ and $g : Y \to Z$ such that f and $g \circ f$ are surjective, then g is surjective. [In other words, either prove that this is true, or find sets X, Y, Z and maps f, g such that $f, g \circ f$ are surjective but such that g is not.]

4. Extra Credit

A. Trivial Credit

A correct response to any one of the following trivia questions is worth 1 point. The total number of points you may gain in part A is 1 point.
A1. What is Farshid's middle name?
A2. What is the color of the nameplate on Farshid's office door?
A3. What are the course numbers of the Math Courses for which Laura is currently registered?

B. Non-trivial Credit

A point-sized bug capable of changing direction instantaneously is sitting on the hood of Dr. Evil's car at noon. Exactly 1 mile down a perfectly straight road and facing the opposite direction is Austin Powers's car. [1] At exactly noon, the two cars head toward each other, each car moving at a constant rate of 10 miles per hour. Yes, Dr. Evil and Austin Powers are playing chicken, but at a relatively safe speed of 10 miles per hour. Meanwhhile, also starting exactly at noon, the bug takes off in the direction of Austin Powers's car at the constant rate of 90 miles per hour, hoping to dissuade the two drivers from their perilous journey. The moment he arrives at Austin Powers's car, the bug instataneously turns around and heads back toward Dr. Evil's car at 90 mph, which, once he reaches it, makes him reverse course once again. The bug continues his shuttle diplomacy zipping back and forth between the two cars at a constant speed but changing direction instantaneously each time he reaches one of the cars ... until the two cars crash and squash the bug. Before his untimely demise, what is the exact distance travelled by the diplomatic and tragic bug?

C. (Highly) Non-trivial Credit

Prove or Disprove: If X is a subset of Y, Y is a subset of Z, and X is equivalent to Z, then X is equivalent to Y. (Here these sets need not be finite).

[1] Dr. Evil's car is an Oldsmobile Cutlass 88; Austin Powers's is a Ford Pinto.