**UMASS AMHERST**
**MATH 131H**
**HONORS CALCULUS I SECTION 2**

SAMPLE FINAL QUESTIONS

1. Determine the derivative of the given function **using the definition of the derivative:**

$$f(x) = \frac{1}{\sqrt{x}}.$$

2. Find all real numbers $a$ such that the tangent to the graph of $y = \ln(x)$ passes through the origin. Show all your work.

3. Prove that $e^x > x$ for all real numbers $x$. (Hint: consider the derivative of $e^x - x$).

4. Let

$$f(x) = \sin(\tan^{-1}(x)).$$

Determine $f'(x)$.

5. Find the following limits

(a)

$$\lim_{x \to 0} \frac{\sin(3x)\sin(5x)}{x^2}$$

(b)

$$\lim_{x \to \infty} \frac{5x^2 + 2x - 3}{3 - x^2}$$

(c)

$$\lim_{x \to 0} \frac{\tan x - x}{x^3}$$

(d)

$$\lim_{x \to 0} \csc x - \cot x$$

(e)

$$\lim_{x \to -\infty} x^2 e^x$$

6. Is the function

$$f(x) = \begin{cases} -x + 5 & \text{if } x < 5 \\ \sin(x - 5) & \text{if } x \geq 5 \end{cases}$$

differentiable everywhere ? Explain.

7. Find all vertical and horizontal asymptotes for $\frac{\cos(x)}{x^3 + 3x^2 + 2x}$.

8. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

9. A cylindrical can without a top is made to contain $V cm^3$ of liquid. Find the dimensions that will minimize the cost of the metal to make the can.
10. Find the absolute maximum of the function $\ln(x^2 + x)/x$ (if it exists). If it doesn't exist, prove that it doesn't exist. Find the intervals over which the function is concave up. Find all inflection points, if any.
11. Give the linear approximation to the function $f(x) = \ln(x)/x$ at $a = e$.
12. A lighthouse is located on a small island 3 km away from the nearest point $P$ on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from $P$?