Math 461 Homework 3

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Reading: Stillwell, Sections 2.7, 2.8.

Justify your answers carefully. Complete proofs are expected (as in MATH 300).

(1) (Parallel Desargues configuration) Let L,M, and N be three lines in the plane all passing through a point O. Let A, A' be points on L, B, B' be points on M, and C, C' be points on N, with the two points on each line being on the same side of O. (We say the two triangles ΔABC and $\Delta A'B'C'$ are *in perspective* from O.) Show that if AB is parallel to $A'B'$ and BC is parallel to $B'C'$ then AC is parallel to $A'C'$.

[Hint: What is Thales' theorem and its converse?]

(2) (Parallel Pappus configuration) Let L and M be two lines in the plane intersecting at a point O. Let A, B, C be points on L and D, E, F be points on M, with the three points on each line being on the same side of O. Show that if AE is parallel to BF and BD is parallel to CE then AD is parallel to CF.

(3) Let ΔABC be a triangle. Let D be the intersection point of the bisector of the angle $\angle BAC$ and the side BC.

(a) Prove that $\text{Area}(\Delta ACD)/\,\text{Area}(\Delta ADB) = |CD|/|DB|$.

(b) Prove that $|CD|/|DB| = |AC|/|AB|$.

[Hint for b: Use the result of HW2Q3a.]

(4) Let A, B, C, D be four points on a circle $\mathcal{C}$ in counterclockwise order. Prove that $\angle DAB + \angle BCD = \pi$.

(5) Let $\mathcal{C}$ be a circle and P a point not lying on $\mathcal{C}$. Let L and M be two lines passing through P such that L intersects $\mathcal{C}$ in two points A and B and M intersects $\mathcal{C}$ in two points C and D. Prove that $|PA| \cdot |PB| = |PC| \cdot |PD|$.

[Hints: What does it mean to say two triangles are similar? Review Section 2.7, Angles in a circle.]

(6) Suppose given a circle $\mathcal{C}$ with center O and a point P outside $\mathcal{C}$. Describe a ruler and compass construction of the two lines through P that are tangent to $\mathcal{C}$.

[Hints: Use the result of HW1Q5. Review Section 2.7, Angles in a circle.]