Math 461 Homework 2

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September 17, 2019

Reading: Stillwell, Sections 1.4, 1.5, 2.3, 2.4, 2.5, and 2.6.

Justify your answers carefully. Complete proofs are expected (as in MATH 300).

(1) We say that a polygon P is *convex* if for any two points A and B in P the line segment AB is contained in P. (Equivalently, P is convex if all the interior angles of P are less than π.)

   (a) Prove that the sum of the interior angles of a convex quadrilateral equals 2π.

   (b) Let $n \geq 3$ be a positive integer. Prove by induction that the sum of the interior angles of a convex n-sided polygon equals $(n-2)\pi$.

(2) Let ΔABC be a triangle such that $|AB| = |AC|$. Suppose given points D on AB and E on AC such that $|BC| = |CD| = |DE| = |EA|$. Determine (with proof) $\angle BAC$.

(3) (a) Suppose given a triangle ΔABC and a point P in the triangle. Let L be the line through P perpendicular to AB and let D be the intersection point of L and AB. Similarly, let M be the line through P perpendicular to AC and let E be the intersection point of M and AC. Show that P lies on the bisector of the angle $\angle BAC$ if and only if $|PD| = |PE|$.

   [Hints: What is Pythagoras' theorem? What is the angle sum of a triangle?]

   (b) Show that the three bisectors of the angles of a triangle are concurrent, that is, they all pass through some point P.

(c) Show that every triangle has an *inscribed circle*: a circle contained in the triangle which is tangent to each of the sides of the triangle. [Hint: Let $\mathcal{C}$ be a circle with center O and P a point on the circle. What is the angle between the radius OP and the tangent to the circle at P? (See HW1Q5.)]

(4) Let ΔABC be a triangle such that $\angle ABC = \pi/2$. Let D be the midpoint of AC. Prove that $|BD| = \frac{1}{2}|AC|$.

(5) (a) Recall that we say two lines L and M in the plane are *parallel* if they do not intersect. Now let L, M, and N be three lines in the plane. Show that if L is parallel to M, M is parallel to N, and L is not equal to N then L is parallel to N.

(b) Consider a quadrilateral $ABCD$ (with vertices A, B, C, D labelled in counter-clockwise order). Let P,Q,R, and S be the midpoints of the sides AB, BC, CD, and DA respectively. Show that the quadrilateral $PQRS$ is a parallelogram.

(6) Let $ABCDEF$ be a convex hexagon (with vertices A, B, C, D, E, F labelled in counter-clockwise order) such that AB is parallel to FC, CD is parallel to BE, and EF is parallel to DA. Show that the two triangles ΔACE and ΔBDF have equal area.