Math 461 Homework 1

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Reading: Stillwell, Sections 1.1, 1.2, 1.3, 2.1, and 2.2.

Justify your answers carefully. Complete proofs are expected (as in MATH 300).

(1) Suppose given two lines L_1 and L_2 intersecting at a point P at angle α, and another line M_1 and a point Q on M_1. Describe a ruler and compass construction of a line M_2 through Q such that the angle between M_1 and M_2 equals α. Prove that your construction is correct.

[Hint: What are the congruence criteria? Given a triangle ΔABC, a point A' and a line L through A', how can we construct a congruent triangle $\Delta A'B'C'$ with base $A'B'$ contained in L?]

(2) Suppose given two triangles ΔABC and $\Delta A'B'C'$ such that $\angle CAB = \angle C'A'B'$, $|AB| = |A'B'|$ and $|BC| = |B'C'|$. Does it follow that the triangles are congruent? Give a proof or a counterexample.

[Hint: Given triangle ΔABC, draw two lines L and M intersecting at a point A' at angle $\angle CAB$, mark a point B' on L such that $|A'B'| = |AB|$, draw the circle $\mathcal{C}$ with center B' and radius $|BC|$, and consider the intersection points of the circle $\mathcal{C}$ with the line M. What happens when $\angle CAB < \angle BCA$?]

(3) Let A,B,C be 3 distinct points in the plane.

(a) Show that the perpendicular bisectors of AB and BC intersect if and only if the points A,B,C do not lie on a line.

[Hint: What is the parallel axiom?]

(b) Prove that if the points A,B,C do not lie on a line then there exists a unique circle passing through the points.

(4) Recall that we say a n-sided polygon is *regular* if all the sides have equal lengths and all the angles are equal. Given a circle with center O and a point P on the circle, describe a ruler and compass construction of a regular n-gon whose vertices lie on the circle in the cases (a) $n = 4$ and (b) $n = 6$. Prove carefully that your construction is correct in each case.

(5) (a) Let C be a circle with center O and P a point on C. Let L be a line passing through P. We say L is *tangent* to C at P if $L \cap C = \{P\}$. Prove that if OP is perpendicular to L then L is tangent to C.

[Hint: What is the contrapositive statement? Suppose L intersects the circle C at another point Q. What can you say about the angle $\angle OPQ$?]

(b) Prove the converse statement: If L is tangent to C then OP is perpendicular to L.