Math 235.4 Midterm 1

Wednesday, October 7, 2009

Name	
Signature	
Student ID #	

- No calculators, notes, or textbooks are allowed.
- Please turn off and put away all cell phones.
- Show all your work and be sure to justify your answers carefully.
- Place a box around your final answer to each question.
- Raise your hand if you have a question.
- Exam time is 50 mins.

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. Let

$$A = \begin{pmatrix} 1 & 2 & -1 & 2 \\ 3 & 8 & 3 & 4 \\ 2 & 6 & 5 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -2 \\ -4 \\ 1 \end{pmatrix}.$$

(a) (8 points) Find all solutions of the equation $A\mathbf{x} = \mathbf{b}$.

(b) (2 points) Does the equation $A\mathbf{x} = \mathbf{c}$ have a solution for every vector $\mathbf{c}$ in $\mathbb{R}^3$? Explain your answer.

2. Consider the system of linear equations

$$\begin{aligned} x - 3y + 2z &= 3 \\ 4x - 9y + 17z &= 6 \\ x - y + 8z &= c \end{aligned}$$

where c is a real number.

(a) (7 points) For what values of c does the system of equations have a solution? Find all solutions in each case.

(b) (3 points) Explain your results from part (a) geometrically.

3. Let

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 5 & 7 \\ 3 & 4 & 1 \end{pmatrix}.$$

(a) (8 points) Compute the inverse A^{-1} of A.

(b) (2 points) Using your answer to part (a), find the solution of the system of linear equations

$$\begin{aligned} x + 2y + 2z &= 1 \\ 2x + 5y + 7z &= 3 \\ 3x + 4y + z &= -1 \end{aligned}$$

4. Find the matrix of the following linear maps.

(a) (2 points) The map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by rotation about the origin through an angle of $3\pi/4$ radians anticlockwise.

(b) (4 points) The map $U\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by projection onto the line through the origin in the direction $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(c) (4 points) The map $V\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by reflection in the plane $2x - y + z = 0$.

5. Let T, U, V be the linear maps from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by the matrices

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}.$$

(a) (4 points) Describe the maps T, U, V geometrically.

(b) (6 points) Compute the matrices of the compositions $T \circ T$, $U \circ U$, and $T^{-1} \circ V \circ T$, and interpret your results geometrically.