Problem $\boxed{1A}$.

$$\int_0^{2\pi} \frac{1}{2}(1+\sin\theta)^2\, d\theta = \frac{1}{2}\int_0^{2\pi} 1+2\sin\theta+\sin^2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} 1+2\sin\theta + \frac{1-\cos 2\theta}{2}\, d\theta$$

$$= \frac{1}{2}\left[\theta - 2\cos\theta + \frac{\theta - \frac{1}{2}\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{1}{2}\left(2\pi \cancel{-2} + \frac{2\pi}{2} - \cancel{(-2)}\right) = \frac{3\pi}{2}$$

✓

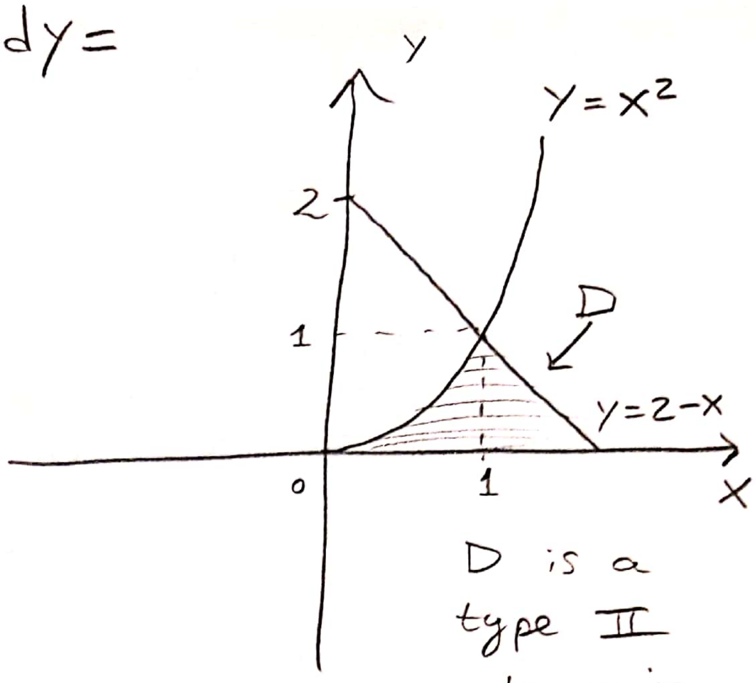
Problem $\boxed{1B}$. $\iint_D xy^2 - x \, dx\,dy =$

$$= \int_0^1 \int_{\sqrt{y}}^{2-y} xy^2 - x \, dx\,dy =$$

$$= \int_0^1 \int_{\sqrt{y}}^{2-y} x(y^2-1)\, dx\,dy =$$

$$= \int_0^1 \left[\frac{1}{2}x^2(y^2-1)\right]_{\sqrt{y}}^{2-y} dy =$$

$$= \int_0^1 \frac{1}{2}(2-y)^2(y^2-1) - \frac{1}{2}y(y^2-1)\, dy =$$



D is a type II domain.

$$= \frac{1}{2}\int_0^1 (y^2-1)\left[(2-y)^2 - y\right] dy = \frac{1}{2}\int_0^1 (y^2-1)(4-4y+y^2-y)\,dy$$

$$= \frac{1}{2}\int_0^1 (y^2-1)(4-5y+y^2)\,dy = \frac{1}{2}\int_0^1 4y^2-5y^3+y^4-4+5y-y^2\,dy$$

$$= \frac{1}{2}\int_0^1 y^4-5y^3+3y^2+5y-4\,dy = \frac{1}{2}\left[\frac{1}{5}y^5-\frac{5}{4}y^4+y^3+\frac{5}{2}y^2-4y\right]_0^1$$

$$= \frac{1}{2}\left(\frac{1}{5}-\frac{5}{4}+1+\frac{5}{2}-4\right) = \frac{1}{2}\,\frac{4-25+20+50-80}{20}$$

$$= \frac{-31}{40} \checkmark$$

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Problem $\boxed{1C}$.

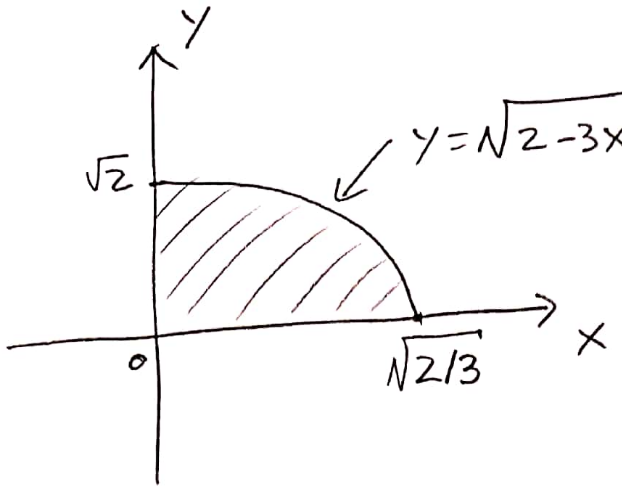
$$\operatorname{curl} \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ z-y & x-z & -x-y \end{vmatrix}$$

$$= \langle -1+1, 1+1, 1+1 \rangle = \langle 0,2,2 \rangle$$

Normalizing $\langle 0,2,2 \rangle$ we obtain $\frac{1}{\sqrt{2^2+2^2}} \langle 0,2,2 \rangle$

$$= \frac{1}{2\sqrt{2}} \langle 0,2,2 \rangle = \frac{1}{\sqrt{2}} \langle 0,1,1 \rangle \quad \checkmark$$

Problem $\boxed{10}$.



$y=\sqrt{2-3x^2} \Rightarrow y^2 = 2-3x^2$

$\Rightarrow y^2-2 = -3x^2$

$\Rightarrow \frac{2-y^2}{3} = x^2$

$\Rightarrow \sqrt{(2-y^2)/3} = x$

$$\int_0^{\sqrt{2/3}} \int_0^{\sqrt{2-3x^2}} \sin(y^2)\, dy\, dx = \int_0^{\sqrt{2}} \int_0^{\sqrt{(2-y^2)/3}} \sin(y^2)\, dx\, dy.$$

✓

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Problem $\boxed{1E}$.

$$\int_0^2\int_0^x\int_{x-y}^{x+y} x\,dz\,dy\,dx = \int_0^2\int_0^x \left[xz\right]_{x-y}^{x+y} dy\,dx$$

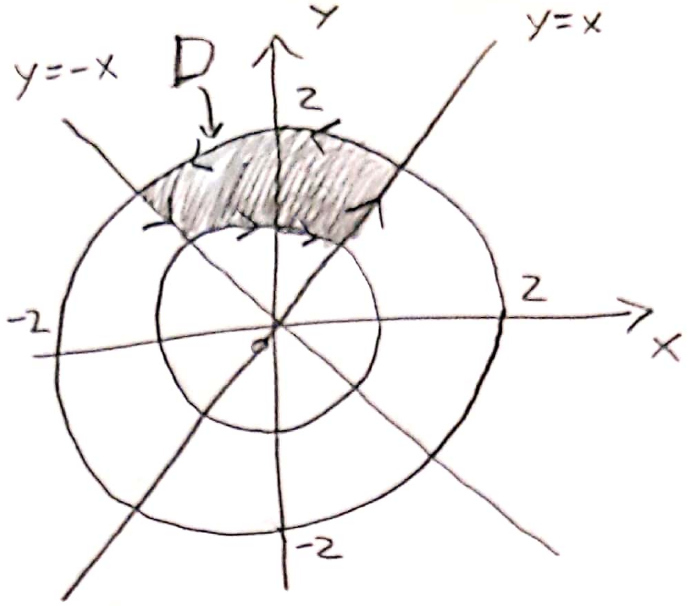
$$= \int_0^2\int_0^x \cancel{x^2}+xy-\cancel{x^2}+xy\,dy\,dx$$

$$= \int_0^2\int_0^x 2xy\,dy\,dx = \int_0^2 \left[xy^2\right]_0^x dx$$

$$= \int_0^2 x^3\,dx = \left[\frac{1}{4}x^4\right]_0^2 = \frac{1}{4}16 = 4.$$

✓

Problem 2



$$\oint_C \overbrace{\left(\sqrt{2+x^3}-8y^3\right)}^{P}dx+\overbrace{\left(8x^3+\sqrt{1+y^3}\right)}^{Q}dy$$

$$= \iint_D 24x^2-\left(-24y^2\right)dA = 24\iint_D x^2+y^2\,dA = 24\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_1^2 r^2 r\,dr\,d\theta$$

$$= 24\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\theta \int_1^2 r^3\,dr = 24\,\frac{\pi}{2}\left[\frac{1}{4}r^4\right]_1^2$$

$$= 12\pi\left(\frac{1}{4}2^4-\frac{1}{4}\right)$$

$$= \cancel{12}^{\,3}\pi\,\frac{2^4-1}{\cancel{4}_{\,1}} = 3\pi(15)$$

$$= 45\pi$$

Problem 3.

$\vec{r}\,'(t) = \langle 1, -1 \rangle$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^2 \langle (1+t)(2-t)^2, -(1+t) \rangle \cdot \langle 1, -1 \rangle \, dt$$

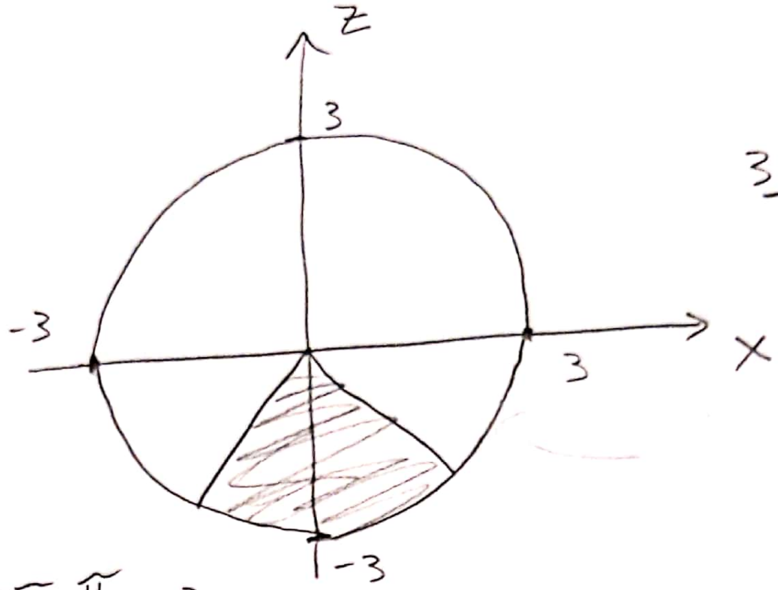
$$= \int_0^2 (1+t)(2-t)^2 + (1+t) \, dt = \int_0^2 (1+t)\left[(2-t)^2 + 1\right] dt$$

$$= \int_0^2 (1+t)\left(4 - 4t + t^2 + 1\right) dt = \int_0^2 (1+t)\left(5 - 4t + t^2\right) dt$$

$$= \int_0^2 5 - 4t + t^2 + 5t - 4t^2 + t^3 \, dt = \int_0^2 5 + t - 3t^2 + t^3 \, dt$$

$$= \left[5t + \frac{1}{2}t^2 - t^3 + \frac{1}{4}t^4\right]_0^2 = 10 + 2 - 8 + 4 = 8.$$

Problem 4.



$0 \le \theta \le 2\pi$
$0 \le \rho \le 3$
$\frac{3\pi}{4} \le \varphi \le \pi$

$$\text{Volume} = \int_0^{2\pi}\int_{\frac{3\pi}{4}}^{\pi}\int_0^3 \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= 2\pi \int_{\frac{3\pi}{4}}^{\pi} \sin\varphi \, d\varphi \int_0^3 \rho^2 \, d\rho$$

$$= 2\pi \left[-\cos\varphi\right]_{\frac{3\pi}{4}}^{\pi} \left[\frac{1}{3}\rho^3\right]_0^3$$

$$= 2\pi \left(1 - \left(-\left(-\frac{\sqrt{2}}{2}\right)\right)\right) 9$$

$$= 18\pi \left(1 - \frac{\sqrt{2}}{2}\right)$$

$$= 9\pi \left(2 - \sqrt{2}\right)$$

Problem 5.

$r_u(u,v) = \langle 1, 1, 2 \rangle$

$r_v(u,v) = \langle 1, -1, 1 \rangle$

$$r_u \times r_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix} = \langle 3, 1, -2 \rangle$$

$$|r_u \times r_v| = \sqrt{9+1+4} = \sqrt{14}$$

$$\iint_S x+y+z \, dS = \int_0^1 \int_0^2 \left(u + \cancel{v} + u - \cancel{v} + 1 + 2u + v\right) \sqrt{14} \, du \, dv$$

$$= \int_0^1 \int_0^2 (4u + v + 1) \sqrt{14} \, du \, dv$$

$$= \sqrt{14} \int_0^1 \left[2u^2 + vu + u\right]_0^2 dv$$

$$= \sqrt{14} \int_0^1 8 + 2v + 2 \, dv = \sqrt{14} \int_0^1 10 + 2v \, dv$$

$$= \sqrt{14} \left[10v + v^2\right]_0^1 = \sqrt{14} \cdot 11$$

Problem 6.

By Stoke's theorem

$$\iint_S \text{curl}\left(xyz\vec{i} + z\vec{j} - y\vec{k}\right) \cdot dS = \oint_C \left(xyz\vec{i} + z\vec{j} - y\vec{k}\right) \cdot d\vec{r} = \star$$

$\vec{r}(t) = \langle 0, \sqrt{2}\cos(t), \sqrt{2}\sin(t) \rangle$, $0 \le t \le 2\pi$

$\vec{r}'(t) = \langle 0, -\sqrt{2}\sin(t), \sqrt{2}\cos(t) \rangle$,

$$\star = \int_0^{2\pi} \left(\sqrt{2}\sin(t)\vec{j} - \sqrt{2}\cos(t)\vec{k}\right) \cdot \left(-\sqrt{2}\sin(t)\vec{j} + \sqrt{2}\cos(t)\vec{k}\right) dt$$

$$= \int_0^{2\pi} -2\sin^2(t) - 2\cos^2(t)\, dt = -2\int_0^{2\pi} dt = -4\pi$$

So $n = -4$.

✓

Problem 7. Let E be the volume inside S.

$$\iint_S \vec{F} \cdot dS = \iiint_E \operatorname{div}(\vec{F})\, dV =$$

spherical coordinates

$$= \frac{1}{4\pi} \iiint_E x^2+y^2+z^2\, dV = \frac{1}{4\pi} \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{10} \rho^2 \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$= \frac{1}{4\pi} 2\pi \int_0^{\frac{\pi}{2}} \sin\varphi\, d\varphi \int_0^{10} \rho^4\, d\rho$$

$$= \frac{1}{2} \Big[-\cos\varphi\Big]_0^{\frac{\pi}{2}} \Big[\frac{1}{5}\rho^5\Big]_0^{10}$$

$$= \frac{1}{2}(1)\frac{1}{5}10^5 = 10^4 = 10000$$

✓