Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Lecturer __________________________ **Section (01, 02, 03, etc.)** _________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 233 **Practice - Final Exam**

DISCLAIMER: This practice exam is intended to give you an idea about what a two-hour final exam is like. It is not possible for any one exam to cover every topic, and the content, coverage and format of your actual exam could be different from this practice exam.

Instructions

- **Turn off all cell phones!**
- **Turn off all cell phones!** Put away all electronic devices such as smartphones, smartwatches, laptops, tablets etc.
- There are seven (7) questions in this exam.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate clearly where your work is for the grader.
- Calculators are **not** allowed, nor are formula sheets or any other materials with helpful information.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Unless indicated otherwise, you must show work to obtain credit for your answers.**
- Be ready to show your UMass ID card when you hand in your exam booklet.
- **By signing my name above, I pledge that I have neither given nor received any aid on this exam.**

1. (20 points) For each question, please select the best response. Please clearly indicate your choice; ambiguous answers will not receive credit. In this problem, there is *no partial credit* awarded and it is *not necessary to show your work.*

(a) (4 points) Find the area enclosed by the polar graph $r = 1 + \sin(\theta)$ where $0 \leq \theta \leq 2\pi$.

(i) 3π (ii) 2π (iii) $\frac{3}{2}\pi$

(iv) $\frac{\pi}{3}$ (v) $\frac{4}{3}\pi$ (vi) π

(b) (4 points) Let D be the region in the xy-plane enclosed by $y = 0$, $y = x^2$, and $y = 2 - x$. Compute the following double integral: $\iint_D (xy^2 - x)dA$

(i) $-\frac{23}{12}$ (ii) $-\frac{19}{20}$ (iii) $-\frac{\sqrt{5}}{3}$

(iv) $-\frac{3}{2}$ (v) $-\frac{31}{40}$ (vi) $-\frac{6}{7}$

(c) (4 points) The expression $(\nabla \times \vec{F}) \cdot \vec{n}$ appearing in Stokes' Theorem is interpreted as the rotation of the vector field $\vec{F}$ about the unit vector $\hat{\vec{n}}$ at (x, y, z). Any unit normal vector $\vec{n}$ that maximizes the rotation of $\vec{F}$ has to be in the same direction as $\nabla \times \vec{F}$.

For the vector field $\vec{F} = \langle z - y, x - z, -x - y\rangle$, what is the unit normal vector $\vec{n}$ that maximizes the rotation?

(i) $\frac{1}{\sqrt{2}}(\vec{i} - \vec{j} + \vec{k})$ (ii) $\langle 0, 2, 2\rangle$ (iii) $\frac{1}{\sqrt{2}}(\vec{j} + \vec{k})$

(iv) $\frac{1}{3}(2\vec{i} + 2\vec{j} - \vec{k})$ (v) $\frac{1}{\sqrt{2}}(\vec{i} + \vec{k})$ (vi) $-\langle 0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\rangle$

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*Continuation of **1.***

(d) (4 points) Switch the order of integration in the following double integral: $\int_0^{\sqrt{2/3}} \int_0^{\sqrt{2-3x^2}} \sin(y^2)dydx$

(i) $\int_0^{\sqrt{2/3}} \int_0^{\sqrt{2-3y^2}} \sin(y^2)dxdy$

(ii) $\int_0^{\sqrt{2}} \int_0^{\sqrt{(2-y^2)/3}} \sin(y^2)dxdy$

(iii) $\int_0^{\sqrt{3}} \int_0^{\sqrt{2-3y^2}} \sin(y^2)dxdy$

(iv) $\int_0^{\sqrt{2}} \int_0^{\sqrt{3-2y^2}} \sin(y^2)dxdy$

(v) $\int_0^{\sqrt{2}} \int_0^{\sqrt{(3-y^2)/2}} \sin(y^2)dxdy$

(vi) $\int_0^{\sqrt{3}} \int_0^{\sqrt{2-3x^2}} \sin(x^2)dxdy$

(e) (4 points) Evaluate the triple integral $\iiint_E x\,dV$ where

$E = \{(x, y, z) \mid 0 \le x \le 2,\ 0 \le y \le x,\ x - y \le z \le x + y\}$.

(i) 4

(ii) $\frac{1}{4}$

(iii) 2

(iv) -2

(v) 16

(vi) 8

2. (20 points) Use Green's Theorem to evaluate the line integral

$$\oint_C (\sqrt{2+x^3} - 8y^3)\,dx + (8x^3 + \sqrt{1+y^3})\,dy$$

where C is the boundary curve of the region
$D = \{(x,y) : 1 \le x^2 + y^2 \le 4, -y \le x \le y, y \ge 0\}$, traversed in a positive sense.

3. (20 points) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = xy^2\,\vec{i} - x\,\vec{j}$ and C is given by $\vec{r}(t) = (1+t)\,\vec{i} + (2-t)\,\vec{j}$ with $0 \le t \le 2$.

4. (20 points) Use spherical coordinates to find the volume of the solid bounded below the surface $z = -\sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 9$.

5. (20 points) Consider the parallelogram S parametrized by

$$\vec{r}(u,v) = \langle u+v, u-v, 1+2u+v \rangle,$$

where $0 \leq u \leq 2$ and $0 \leq v \leq 1$. Compute the surface integral $\iint_S (x+y+z)dS$.

6. (20 points) Let C be the circle $y^2 + z^2 = 2$, where $x = 0$, oriented counter-clockwise when viewed from the positive x axis, and let S be any oriented capping surface of C such that C is positively oriented with respect to the surface S. Let the vector field $\vec{F}$ be defined as $\nabla \times (xyz\vec{\imath} + z\vec{\jmath} - y\vec{k})$. Then the flux of $\vec{F}$ through such a surface S is a multiple of π, that is, $\iint_S \nabla \times (xyz\vec{\imath} + z\vec{\jmath} - y\vec{k}) \cdot d\vec{S} = n\pi$ for some number n. What is the number n?

7. (20 points) Let $\vec{F}(x,y,z) = \frac{1}{12\pi}\langle x^3, y^3, z^3\rangle$, and let S be the surface consisting of the upper hemisphere ($z \geq 0$) of $x^2+y^2+z^2=100$, together with the radius 10 disk in the xy plane, centered at the origin, and orient S with an outward normal. Calculate the flux $\displaystyle\iint_S \vec{F}\cdot\vec{n}\,dS$.

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