
DISCLAIMER: This practice exam is intended to give you an idea about what a two-hour final exam is like. It is not possible for any practice exam to cover every topic, and the content, coverage and format of your actual exam could be different from this practice exam.

Part I: Multiple Choice Problems. You only need to give the answer; no justification needed.

1. Determine which of the following vector fields is conservative.

   (a) $\langle -y, x, 0\rangle$ (b) $\langle xy, yz, zx\rangle$ (c) $\langle 2xy, x^2+2yz, y^2\rangle$

   (d) $\langle y^2, z^2, x^2\rangle$ (e) $\langle x^2y, y^2z, z^2x\rangle$ (f) $\langle x, x, x\rangle$

2. Let $z = x^2y$ and let x, y be functions of t with $x(1) = 1$, $y(1) = 2$, $x(2) = 3$, $y(2) = 4$, $x'(1) = A$, $y'(1) = B$, $x'(2) = C$, $y'(2) = D$. Find $\frac{dz}{dt}$ when $t = 1$.

   (a) 4A+D (b) 4A+B (c) 4C+D (d) A+2D (e) 4C+2D (f) A+4B

3. Find the minimum speed of the particle whose position function is $\vec{r}(t) = \langle t^2, 1-2t, t\rangle$.

   (a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$ (e) $\sqrt{5}$ (f) 6

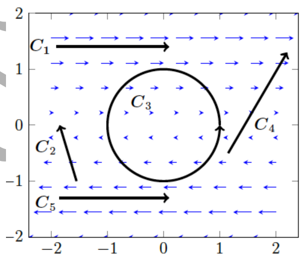
4. Find the value of the gradient vector field of the function $z = x^2y^3$ at the point $(1, 1)$.

   (a) $\langle 2, -3\rangle$ (b) $\langle 2, 3\rangle$ (c) $\frac{1}{\sqrt{11}}\langle 3, -2\rangle$

   (d) $\langle 2, -3, 1\rangle$ (e) $\frac{1}{\sqrt{11}}\langle 2, 3\rangle$ (f) $\frac{1}{\sqrt{14}}\langle 3, -2, 1\rangle$

5. The figure depicts a 2-dimensional vector field $\vec{F}$ as well as five oriented paths. Place in order, from smallest to largest, the five values.

   $A = \int_{C_1} \vec{F}\cdot d\vec{r}$ $B = \int_{C_2} \vec{F}\cdot d\vec{r}$ $C = \int_{C_3} \vec{F}\cdot d\vec{r}$ $D = \int_{C_4} \vec{F}\cdot d\vec{r}$ $E = \int_{C_5} \vec{F}\cdot d\vec{r}$



   smallest ☐ ☐ ☐ ☐ ☐ largest

6. Find a non-zero vector perpendicular to the plane that passes through the points $(1, 0, 0)$, $(5, 4, 0)$ and $(0, 4, 1)$.

   (a) $\langle 1, -4, -5\rangle$ (b) $\langle 1, -4, 5\rangle$ (c) $\langle -5, 1, 4\rangle$

   (d) $\langle 4, -4, 20\rangle$ (e) $\langle 4, 1, 5\rangle$ (f) $\langle 4, 4, 20\rangle$

PART II: Written Problems. To earn full credit for the following problems you must show your work. You can leave answers in terms of fractions and square roots.

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1. Find an equation for the plane consisting of all points that are equal distance from the two points $(1, 1, 0)$ and $(0, 1, 1)$

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2. Find the maximum and minimum values of the function $f(x, y) = x^2 - y^2$ subject to the constraint $x^2 + y^2 \leq 1$.

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3. Consider the 2-dimensional vector field $\vec{F}(x, y) = e^y \sin x\vec{i} + e^y \cos x\vec{j}$.
   (a) Is this vector field conservative?
   (b) Compute the work done by $\vec{F}$ along the line segment from $(0, 0)$ to $(\pi, 1)$.

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4. Determine the flux of the vector field $\vec{F} = ye^{z^2}\vec{i} + y^2\vec{j} + e^{xy}\vec{k}$ across the cylinder S defined by $x^2 + y^2 = 9$ and $0 \leq z \leq 4$, with positive orientation.

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5. Evaluate $\displaystyle\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = \langle x^2z, xy^2, z^2\rangle$ and C is the curve of intersection of the plane $x+y+z = 1$ with the cylinder $x^2 + y^2 = 9$, oriented counterclockwise as viewed from above.