UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 233 **PRACTICE EXAM 2**

Last Name: ______________ **First Name:** ______________

Spire ID: ______________ **Section:** ______________

Honesty Pledge: By signing my name below, I pledge that I have neither given nor received any aid on this exam.

Signature: ______________

INSTRUCTIONS

- This test booklet should **not** be opened until the proctors ask you to do so.
- **All cell phones must be turned off!** Put away all electronic devices such as smartphones, smartwatches, laptops, tablets, calculators etc.
- Calculators, formula sheets or any materials with helpful information are **not** allowed.
- This exam contains **??** pages (including this cover page) and **??** questions. Check to see if any pages are missing. Complete all requested information on the top of this page.
- Do all work in this exam booklet. Make sure to label any additional work done on the blank sheets by its question number, and indicate on the relevant question where additional work can be found.
- **Organize your work in an unambiguous order. Unless indicated otherwise, you must show ALL work to obtain credit for your answers. Unsupported answers will not receive credit. Please write clearly; answers deemed illegible will not be graded. Reduce your answers to their simplest form and box your answers.**
- Be ready to show your UMass ID card when you hand in your exam booklet.

Question 1 (20 points)
For each question, **fill in** the correct response. Please clearly indicate your choice; ambiguous answers will not receive credit. In this problem, there is *no partial credit awarded* and it is *not necessary to show your work*.

(a) (4 points) Evaluate the iterated integral $\int_0^2 \int_y^{2y} 2xy \, dx \, dy$

○ 12 ○ 4 ○ 2

○ 3 ○ 5 ○ 7

(b) (4 points) Let R be the region of a unit circle centered at the origin. The limits of integration a, b of $\iint_R f(x,y)\,dA = \int_{-1}^{1} \int_a^b f(x,y)\,dx\,dy$ are:

○ $a = -1,\ b = 1$ ○ $a = 0,\ b = 1$

○ $a = 0,\ b = \sqrt{1-y^2}$ ○ $a = -\sqrt{1-y^2},\ b = \sqrt{1-y^2}$

○ $a = -\sqrt{1-x^2},\ b = \sqrt{1-x^2}$ ○ $a = 0,\ b = 2\pi$

(c) (4 points) Estimate the volume of the solid that lies above the square $R = [0,2] \times [0,2]$ and below the surface $z = 16 - x^2 - 2y^2$, by dividing the square in four equal squares and choosing as sample point, for each of the four squares, the point on the upper right corner.

○ 28 ○ 25 ○ 52

○ 43 ○ 34 ○ 48

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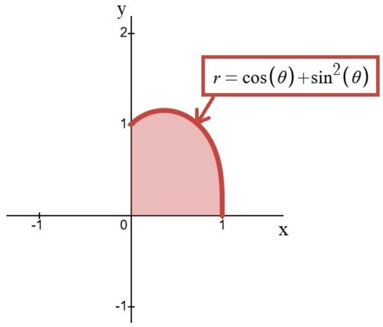
Continuation of question **1.**

(d) (4 points) Which integral represents the area of the region in the figure? (The polar function in the box describes the boundary.)



○ $\int_0^{\pi/2} \int_0^1 (\cos(\theta) + \sin^2(\theta)) r dr\, d\theta$

○ $\int_0^{\pi} \int_0^{\cos(\theta)+\sin^2(\theta)} dr\, d\theta$

○ $\int_0^{\pi/2} \int_0^1 r dr\, d\theta$

○ $\int_0^{\pi/2} \int_0^1 dr\, d\theta$

○ $\int_0^{\pi/2} \int_0^{\cos(\theta)+\sin^2(\theta)} r dr\, d\theta$

○ $\int_0^{\pi} \int_0^1 r dr\, d\theta$

(e) A certain triple integral $\iiint_P f(x,y,z)\, dV$ over a solid pyramidal region P in the first octant can be expressed as the iterated integral

$$\int_0^1 \int_0^{1-x} \int_0^{1-z} f(x,y,z)\, dy\, dz\, dx\,.$$

Which of the following represents the same triple integral $\iiint_P f(x,y,z)\, dV$?

○ $\int_0^1 \int_0^{1-x} \int_0^{1-z-x} f(x,y,z)\, dy\, dx\, dz$

○ $\int_0^1 \int_0^{1-z} \int_0^{1-z} f(x,y,z)\, dx\, dy\, dz$

○ $\int_0^1 \int_y^1 \int_{1-x}^{1-y} f(x,y,z)\, dz\, dx\, dy$

○ $\int_0^1 \int_y^{1-y} \int_0^{1-z} f(x,y,z)\, dy\, dz\, dx$

○ $\int_0^1 \int_0^{y} \int_0^{1-z} f(x,y,z)\, dx\, dz\, dy$

○ $\int_0^1 \int_0^{1-z} \int_0^{1-y} f(x,y,z)\, dx\, dy\, dz$

Question 2 (15 points)

Find and classify all critical points of the function $f(x,y) = 4x^3 - 6x^2y - 3y^2 + 12y - 7$ as local maxima, local minima or saddle points.

Question 3 (15 points)

Parts (a) and (b) are distinct from each other.

(a) Evaluate the integral $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{xe^{x^2+y^2}}{\sqrt{x^2+y^2}}\,dy\,dx$.

(b) Let $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{x}{y^5+1}\,dy\,dx$. Sketch the region of integration. Reverse the order of integration and evaluate the integral.

Question 4 (15 points)

Parts (a) and (b) are distinct from each other.

(a) Let S be the portion of the cylinder $z = \sqrt{4 - x^2}$ cut out by the planes $z = 0$, $y = 0$, and $y = x$, in the first octant (positive octant). Set up and evaluate a double integral to calculate the surface area of S.

(b) Find the maximum and minimum values of $f(x, y) = xy$ on the ellipse $4x^2+y^2 = 16$.

Question 5 (15 points)

Parts (a) and (b) are distinct from each other.

(a) Let H be the hemispherical solid region described by $x^2 + y^2 + z^2 \leqslant 4, z \geqslant 0$. Evaluate $\iiint_H 3\sqrt{x^2 + y^2 + z^2}\, dV$.

(b) Use cylindrical coordinates to compute the integral of the function $f(x, y, z) = z$ over the solid region bounded by the cone $z = \sqrt{x^2 + y^2}$ and the paraboloid $z = x^2 + y^2$.

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