Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Lecturer _________________________ **Section (01, 02, 03, etc.)** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 233 **Practice - Exam 2**

Instructions

- **Turn off all cell phones!** Put away all electronic devices such as smartphones, smartwatches, laptops, tablets etc.
- There are six (6) questions in this exam.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate clearly where your work is for the grader.
- Calculators are **not** allowed, nor are formula sheets or any other materials with helpful information.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Unless indicated otherwise, you must show work to obtain credit for your answers.**
- Be ready to show your UMass ID card when you hand in your exam booklet.
- **By signing my name above, I pledge that I have neither given nor received any aid on this exam.**

QUESTION	POINTS	SCORE
1	20	
2	15	
3	15	
4	15	
5	15	
6	20	
TOTAL	100	

1. (20 points) For each question, please select the best response. Please clearly indicate your choice; ambiguous answers will not receive credit. In this problem, there is *no partial credit* awarded and it is *not necessary to show your work.*

(a) (4 points) The area of the region enclosed by the polar graph $r = 2\sin\theta$ is

(i) $1/\sqrt{2}$	(ii) $\pi/\sqrt{2}$	(iii) 1
(iv) $\pi/2$	(v) π	(vi) 2π

(b) (4 points) Evaluate the iterated integral $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

(i) $\frac{\pi}{2}$	(ii) $\frac{\sqrt{3}}{2}$	(iii) $\frac{\pi}{6}$
(iv) $\frac{5}{2}$	(v) $\frac{1}{2}$	(vi) $\frac{2}{\sqrt{\pi}}$

(c) (4 points) Find the critical points of the function: $f(x,y) = \frac{2}{3}x^3 + \frac{1}{3}y^3 - xy.$

(i) $(0,0), (1,0)$	(ii) $(1,2), (4^{\frac{1}{3}}, 2\cdot 4^{\frac{2}{3}})$	(iii) $(1,1), (2,2)$
(iv) $(0,1), (4^{-\frac{1}{3}}, 2\cdot 4^{-\frac{3}{2}})$	(v) $(1,1), (\sqrt{2}, \sqrt[3]{2})$	(vi) $(0,0), (4^{-\frac{1}{3}}, 2\cdot 4^{-\frac{2}{3}})$

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Continuation of ***1.***

(d) (4 points) Set up the double integral $\iint_R f(x,y)\,dA$ over the shaded region R shown in Figure **??** in the order $dy\,dx$. (The region is bounded by $x=1$, $y=1-x^2$, and $y=e^x$).

(i)	$\int_0^1\int_{x^2}^{\ln x} f(x,y)\,dy\,dx$	(ii)	$\int_0^1\int_{1-x^2}^{\ln x} f(x,y)\,dy\,dx$
(iii)	$\int_1^0\int_{1-x^2}^{e^x} f(x,y)\,dy\,dx$	(iv)	$\int_1^0\int_{e^x}^{1-x^2} f(x,y)\,dy\,dx$
(v)	$\int_0^1\int_{1-x^2}^{e^x} f(x,y)\,dy\,dx$	(vi)	$\int_0^1\int_{e^x}^{1-x^2} f(x,y)\,dy\,dx$

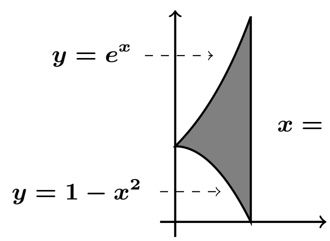

Figure 1: The region R from 1**??**

(e) (4 points) Let E be the solid region bounded by the paraboloid $z = 2 + x^2 + y^2$, the cylinder $x^2+y^2=1$, and the xy-plane (Figure **??**). In cylindrical coordinates, when written as an iterated integral the triple integral $\iiint_E e^z\,dV$ becomes

(i)	$\int_0^{2\pi}\int_0^1\int_0^{2+r^2} re^z\,dr\,d\theta\,dz$	(ii)	$\int_0^{2\pi}\int_0^1\int_0^{2+r^2} re^z\,d\theta\,dz\,dr$
(iii)	$\int_0^{2\pi}\int_0^1\int_0^{2+r^2} re^z\,dz\,dr\,d\theta$	(iv)	$\int_0^{2\pi}\int_0^2\int_0^{2+r^2} r^2e^z\,dr\,d\theta\,dz$
(v)	$\int_0^{2\pi}\int_0^2\int_0^{2+r^2} r^2e^z\,d\theta\,dz\,dr$	(vi)	$\int_0^{2\pi}\int_0^2\int_0^{2+r^2} r^2e^z\,dz\,dr\,d\theta$

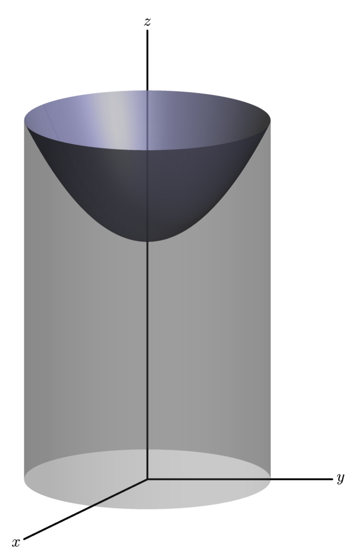

Figure 2: The region E from 1(e)

2. (15 points) Let $f(x, y) = x + y$ and let E be the ellipse

$$x^2 + \frac{y^2}{8} = 1.$$

Find the minimum and maximum value of f on E.

3. (15 points) Let R be the triangular region in the xy-plane with vertices $(0, 0)$, $(1, 1)$, and $(1, 0)$. Find the volume over R and under the paraboloid $z = 2 - x^2 - y^2$.

4. (15 points) Find the surface area of the part of the graph of $z = 3 + 2y + x^4/4$ that lies over the region R in the xy-plane bounded by $y = x^5$, $x = 1$, and the x-axis.

5. (15 points) Let E be the solid region bounded by the unit sphere $x^2+y^2+z^2=1$ and inside the cone $z=\sqrt{x^2+y^2}$. Evaluate $\iiint_E z\,dV$.

6. (20 points) Find the x and y coordinates of all critical points of the function

$$f(x, y) = 2x^3 - 6x^2 + xy^2 + y^2$$

and use the Second Derivative Test to classify them as local minima, local maxima or saddle points.

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