Math 233
2025 Spring

Midterm Exam2 Review2

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1. Find all the critical points of $f(x,y) = x^2y - y^2 - 2y - x^2$ and classify them as local max, local min or saddle points.
2. Find the point on the cone $x^2 + y^2 = z^2$ that is closest to the point $(4,2,0)$.
3. Find the max and min of $f(x,y) = e^{xy}$ over the region $x^2 + 4y^2 \leq 2$.
4. Use upper right Riemann sum with 4 squares ($\Delta x = \Delta y = \pi/2$) to approximate $\int\int_R \cos(x)\sin(y)\,\mathrm{d}A$ with $R = [-\pi/2, \pi/2] \times [0, \pi]$.
5. Evaluate the integral $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\,\mathrm{d}y\,\mathrm{d}x$.
6. Rewrite the integral $\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} x\sin((x^2+y^2)^{3/2})\,\mathrm{d}y\,\mathrm{d}x$ using polar coordinates.
7. Find the area of the region enclosed by $r = \cos(2\theta)$.
8. Let D be the region inside $x^2 + y^2 = 1$, lies above $y = -x$ and below $y = x$. Find the surface area of $z = \sqrt{x^2 + y^2}$ above D.
9. Find the area of the part of the sphere $x^2+y^2+z^2 = 4z$ that lies inside $z = x^2+y^2$.
10. Compute the triple integral of $f(x,y,z) = z$ in the region bounded by $x \geq 0, z \geq 0, y \geq 3x, y^2 + z^2 \leq 9$.
11. Express the solid enclosed by $y = 0$ and $y = 4 - x^2 - 4z^2$ in the order of $\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$ and $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.
12. Find the volume of the solid enclosed by $z = x^2 + y^2$ and $z = 4 - x^2 - y^2$.
13. Evaluate $\int\int\int_R (x^2 + y^2)\,\mathrm{d}V$ with R the region between $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.
14. Rewrite $\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$ in spherical coordinates.