UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 233 **EXAM 1** **Practice Exam - S3**

Last Name: ____________________ **First Name:** ____________________

Spire ID: ____________________ **Section:** ____________________

INSTRUCTIONS

- This test booklet should **not** be opened until the proctors ask you to do so.
- **All cell phones must be turned off!** Put away all electronic devices such as smartphones, smart-watches, laptops, tablets, calculators etc.
- Calculators, formula sheets or any materials with helpful information are **not** allowed.
- This exam contains 8 pages (including this cover page) and 5 questions. Check to see if any pages are missing. Complete all requested information on the top of this page.
- Do all work in this exam booklet. Make sure to label any additional work done on the blank sheets by its question number, and indicate on the relevant question where additional work can be found.
- **Organize your work in an unambiguous order. Unless indicated otherwise, you must show ALL work to obtain credit for your answers. Unsupported answers will not receive credit. Please write clearly; answers deemed illegible will not be graded. Reduce your answers to their simplest form and box your answers.**
- Be ready to show your UMass ID card when you hand in your exam booklet.

Question 1 (20 points)

For each question, **fill in** the correct response. Please clearly indicate your choice; ambiguous answers will not receive credit. In this problem, there is *no partial credit awarded* and it is *not necessary to show your work*.

(a) (4 points) Find the area of the triangle with vertices $(a, 0, 0)$, $(0, 2a, 0)$ and $(0, 0, 3a)$.

- ○ $\frac{3a^2}{2}$
- $\boxed{○\ \frac{7a^2}{2}}$
- ○ $\frac{5a^2}{2}$
- ○ $5a^2$
- ○ $6a^2$
- ○ $7a^2$

(b) (4 points) A vector function $\vec{r}(t)$ representing the curve of intersection between the surfaces $x^2 + y^2 = 1$ and $z = xy^2$ is given by:

- ○ $\langle \sin(t), \cos(t), \cos(t)\sin^2(t) \rangle$
- ○ $\langle \cos^2(t), \sin^2(t), \cos^2(t)\sin^4(t) \rangle$
- ○ $\langle \cos(t), \sin(t), \sin(t)\cos^2(t) \rangle$
- ○ $\langle \sin^2(t), \cos^2(t), \cos^2(t)\sin^4(t) \rangle$
- $\boxed{○\ \langle \cos(t), \sin(t), \cos(t)\sin^2(t) \rangle}$
- ○ $\langle \sin^2(t), \cos^2(t), \sin^2(t)\cos^4(t) \rangle$

(c) (4 points) Assuming $x > 0$, traces of the surface $16x = 4y^2 + z^2$ are:

- ○ hyperbolas or parabolas
- ○ circles or ellipses
- ○ ellipses or hyperbolas
- ○ ellipses, hyperbolas or line pairs
- ○ circles or hyperbolas
- $\boxed{\text{○ ellipses or parabolas}}$

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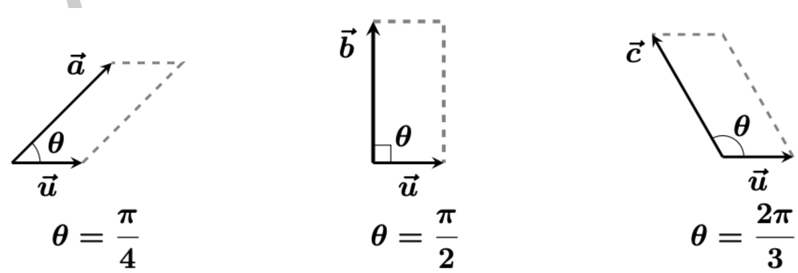
Continuation of question **1.**

(d) (4 points) At a certain instant, the base of a rectangle is 4 cm and increasing at a rate of 3 cm/hr and the height is 6 cm and decreasing at a rate of 2 cm/hr. At what rate is the area of the rectangle changing at the same instant, in units of cm^2/hr?

- ○ 26
- ○ 0
- ○ 14
- ○ $\boxed{10}$
- ○ 24
- ○ 17

(e) (4 points) In the figure below consider $|\vec{a}| = |\vec{b}| = |\vec{c}|$. $\vec{u}$ is a unit vector. Use the parallelograms below to rank $|\vec{a}\times\vec{u}|$, $|\vec{b}\times\vec{u}|$ and $|\vec{c}\times\vec{u}|$ from smallest to largest.

- ○ $|\vec{a}\times\vec{u}| \le |\vec{b}\times\vec{u}| \le |\vec{c}\times\vec{u}|$
- ○ $|\vec{b}\times\vec{u}| \le |\vec{c}\times\vec{u}| \le |\vec{a}\times\vec{u}|$
- ○ $\boxed{|\vec{a}\times\vec{u}| \le |\vec{c}\times\vec{u}| \le |\vec{b}\times\vec{u}|}$
- ○ $|\vec{c}\times\vec{u}| \le |\vec{a}\times\vec{u}| \le |\vec{b}\times\vec{u}|$
- ○ $|\vec{b}\times\vec{u}| \le |\vec{a}\times\vec{u}| \le |\vec{c}\times\vec{u}|$
- ○ $|\vec{c}\times\vec{u}| \le |\vec{b}\times\vec{u}| \le |\vec{a}\times\vec{u}|$



Question 2 (15 points)

(a) Let P be a plane containing the points $(0, 2, 2)$ and $(4, 2, 4)$ that is perpendicular to the plane given by the equation $-2x - 2y + z = 8$. Find the equation of P and express it in the form $ax + by + cz + d = 0$.

$\boxed{x - 2y - 2z + 8 = 0}$

(b) Do the lines $L_1 : x = 5 + t,\ y = 2 - 2t,\ z = -1 - 3t$ and $L_2 : x = 2 + s,\ y = 2s,\ z = 4 - s$ intersect? If so, where do they intersect?

$\boxed{(4, 4, 2)}$

(c) Find the *cosine* of the angle between the following two planes: $ax + by + cz = d$ and $x + 4y + 8z = 5$, where a, b, c, d are constants.

$\boxed{\dfrac{a + 4b + 8c}{9\sqrt{a^2 + b^2 + c^2}}}$

Question 3 (15 points)

(a) The velocity vector of a particle is given by $\vec{v}(t) = t\cos(t^2)\vec{i} + t\sin(t^2)\vec{j} + \frac{1}{1+t}\vec{k}$ for $t \geq 0$. The particle is initially found at the point $(-\frac{1}{2}, 1, 2)$ when $t = 0$. Find the position function of the particle.

$$\boxed{\vec{r}(t) = \frac{1}{2}(\sin(t^2) - 1)\vec{i} - \frac{1}{2}(\cos(t^2) - 3)\vec{j} + (\ln(t+1) + 2)\vec{k}}$$

(b) Compute the length of the curve $\vec{c}(t) = \left\langle \cos(t), \sin(t), \frac{2}{3}t^{\frac{3}{2}} \right\rangle$, where $0 \leq t \leq 3$.

$$\boxed{\frac{14}{3}}$$

Question 4 (15 points)

(a) Find the linear approximation $L(x, y)$ of the function $g(x, y) = \sqrt{2x + 3y} - \dfrac{x}{y}$ at the point $(3, 1)$.

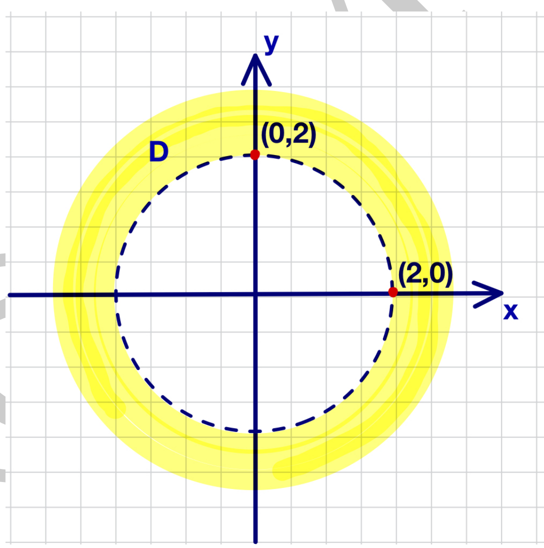
$$\boxed{L(x, y) = -\frac{2}{3}x + \frac{7}{2}y - \frac{3}{2}}$$

(b) Let $z = f(x, y)$ and $x^2y^3 + z = z\cos(z)$. Use implicit differentiation to find $\dfrac{\partial z}{\partial y}$.

$$\boxed{\frac{\partial z}{\partial y} = \frac{3x^2y^2}{\cos(z) - z\sin(z) - 1}}$$

(c) Find and sketch the domain of the function $h(x, y) = \ln(x^2 + y^2 - 4)$.

$$\boxed{D = \{(x, y) |\ x^2 + y^2 > 4\}}$$



Question 5 (15 points)

For parts (a) and (b) let $V(x, y, z) = 2x^2 - 2xy + xyz$ be the electrical potential over a certain region in space, and P be a point with coordinates (1,2,3).

(a) Find the directional derivative of $V(x, y, z)$ at the point P in the direction of the vector $\langle 0, -4, 3 \rangle$.

$\boxed{\frac{2}{5}}$

(b) In which direction does V increase most rapidly at P? What is the value of the maximum rate of increase at P?

$\boxed{\langle 6, 1, 2 \rangle, \quad \sqrt{41}}$

(c) Let $z = xe^y$, where $x = s^2 + t^2$, $y = s^2 - t^2$. Use the chain rule to find $\dfrac{\partial z}{\partial t}$ in terms of *only* t and s.

$\boxed{\frac{\partial z}{\partial t} = 2te^{s^2-t^2}(1 - s^2 - t^2)}$