Last Name ____________________ **First Name**____________________

ID # ____________________ **Signature** ______________________________

Lecturer ______________________________ **Section** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 233 **Exam 1** **Practice Exam - S2**

Instructions

- **Turn off all cell phones!** Put away all electronic devices such as smartphones, smartwatches, laptops, tablets etc.
- There are five (5) questions in this exam.
- Do all work in this exam booklet. You may continue work to the blank page at the end, but if you do so indicate clearly where your work is for the grader.
- Calculators are **not** allowed, nor are formula sheets or any other materials with helpful information.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Unless indicated otherwise, you must show ALL work to obtain credit for your answers. Please write clearly, reduce your answers to their simplest form and box your answers.**
- Be ready to show your UMass ID card when you hand in your exam booklet.
- **By signing my name above, I pledge that I have neither given nor received any aid on this exam.**

QUESTION	POINTS	SCORE
1	20	
2	15	
3	15	
4	15	
5	15	
TOTAL	80	

1. (20 points) For each question, select the best response. Please clearly indicate your choice; ambiguous answers will not receive credit. In this problem, there is *no partial credit* awarded and it is *not necessary to show your work.*

(a) (4 points) For what values of t are the vectors $\vec{u} = \langle t+2, t, t\rangle$ and $\vec{w} = \langle t-2, t+1, 1\rangle$ orthogonal?

(i)	$t=-1, t=2$	(ii)	$t=0, t=3$	(iii)	$\boxed{t=1, t=-2}$
(iv)	$t=2, t=4$	(v)	$t=-3, t=5$	(vi)	$t=0, t=5$

(b) (4 points) A vector function, $\vec{r}(t)$, representing the curve of intersection between the cylinder $x^2+y^2=1$ and the plane $y-z=1$ is given by:

(i)	$\langle \cos^2(t), \sin^2(t), \sin^2(t)-1\rangle$	(ii)	$\langle \sin(t), \cos(t), \sin(t)+1\rangle$
(iii)	$\langle \cos(t), \sin(t), \sin(t)+1\rangle$	(iv)	$\boxed{\langle \cos(t), \sin(t), \sin(t)-1\rangle}$
(v)	$\langle \sin^2(t), \cos^2(t), \cos^2(t)-1\rangle$	(vi)	None of the above

(c) (4 points) Let $z = f(x,y)$, where f is differentiable, $x = g(t)$, $y = h(t)$, $g(1) = 3$, $h(1) = 4$, $g'(1) = -2$, $h'(1) = 5$, $f_x(3,4) = 7$ and $f_y(3,4) = 6$. Find $\dfrac{dz}{dt}$ when $t = 1$.

(i)	18	(ii)	23	(iii)	44
(iv)	32	(v)	13	(vi)	$\boxed{16}$

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Continuation of question **1.**

(d) (4 points) Compute the volume of the parallelepiped formed by the vectors $\vec{a} = \langle 1, 0, 2 \rangle$, $\vec{b} = \langle 2, -1, 0 \rangle$ and $\vec{c} = \langle 4, 1, 1 \rangle$.

(i) 15 (ii) 14 (iii) 13

(iv) $\boxed{11}$ (v) 10 (vi) 9

(e) (4 points) Identify the surface represented by the equation $x^2 + 2y^2 + 3z^2 = 4$.

(i) Sphere. (ii) $\boxed{\text{Ellipsoid.}}$

(iii) Paraboloid. (iv) Hyperboloid of one sheet.

(v) Hyperboloid of two sheets. (vi) Cone.

2. (15 points) Consider the following lines:

$$L_1 : x = 1 + t,\ y = 1 - t,\ z = 3t$$
$$L_2 : x = 7 - 2s,\ y = 4 - s,\ z = 15 - 5s$$

(a) Do these lines intersect? If so, find the point(s) of intersection of L_1 and L_2.

$$\boxed{(1,1,0)}$$

(b) Find the cosine of the *acute* angle between L_1 and L_2.

$$\boxed{\frac{16}{\sqrt{11}\sqrt{30}}}$$

(c) Find a linear equation of the plane containing L_1 and L_2.

$$\boxed{8x - y - 3z - 7 = 0}$$

3. (15 points)
(a) Find the partial derivatives f_x, f_y, f_{xx}, f_{yy} and f_{xy} of the two variable function $f(x, y) = x\sin(x^2y)$.

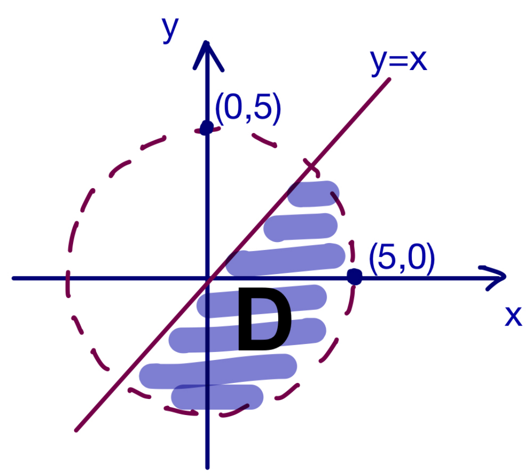
$$f_x = \sin(x^2y) + 2x^2y\cos(x^2y),\ f_y = x^3\cos(x^2y),\ f_{xx} = 6xy\cos(x^2y) - 4x^3y^2\sin(x^2y)$$

$$f_{yy} = -x^5\sin(x^2y),\ f_{xy} = f_{yx} = 3x^3\cos(x^2y) - 2x^4y\sin(x^2y)$$

(b) Find and sketch the domain D of the function

$$f(x, y) = \ln(25 - x^2 - y^2) + \sqrt{x - y}$$

$$D = \{(x, y)|x^2 + y^2 < 25,\ y \leq x\}$$



4. (15 points)

(a) Find a linear equation of the tangent plane to the graph of the function $f(x, y) = \ln(1 + xy)$ at the point $(1, 2, \ln 3)$.

$$\boxed{z = \frac{2}{3}x + \frac{1}{3}y - \frac{4}{3} + \ln(3)}$$

(b) Using your answer from question (a), find the linearization $L(x, y)$ of f at $(1, 2)$ and use it to approximate the value of $f(1.1, 1.9)$. Simplify your answer.

$$\boxed{L(1.1, 1.9) = \frac{1}{30} + \ln(3)}$$

5. (15 points)
(a) Find the position of a particle at time $t = 1$, if the acceleration at time t is given by $\vec{a}(t) = \langle 2t, 0, 3t^2 \rangle$, the initial velocity is $\vec{v}(0) = \langle 1, -1, 0 \rangle$ and the initial position is $\vec{r}(0) = \langle 0, 0, 1 \rangle$.

$$\boxed{\left(\frac{4}{3}, -1, \frac{5}{4}\right)}$$

(b) Find the length of the curve $\vec{r}(t) = \langle \cos^3(t), \sin^3(t) \rangle$, where $0 \leq t \leq \frac{\pi}{2}$.

$$\boxed{\frac{3}{2}}$$

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